Math 164: Homework #5, due on Wednesday, May 6

[**1**] Consider the linear program in problem [5] from previous homework.

(a) Represent the point $x = (6, 12)^T$ as a convex combination of extreme points, plus if applicable, a direction of unboundedness.

(b) Show by the method of your choice that this problem has no finite optimal solution.

[**2**] Consider a linear program with the constraints in standard form

$$Ax = b \text{ and } x \geq \vec{0}.$$

(a) Prove that, if d is a direction of unboundedness for these constraints, then $-d$ cannot be a direction of unboundedness.

(b) Let $\{d_1, ..., d_k\}$ be directions of unboundedness for these constraints. Prove that a nonzero vector $d = \sum_{i=1}^{k} \alpha_i d_i$, with $\alpha_i \geq 0$ is also a direction of unboundedness.

[**3**] Suppose that a linear program in standard form, with bounded feasible region, has l optimal extreme points $\{v_1, v_2, ..., v_l\}$. Prove that a point is optimal for the linear program if, and only if, it can be expressed as a convex combination of $\{v_1, v_2, ..., v_l\}$.

[**4**] Solve the following linear program using the simplex method (graph the feasible region, and outline the progress of the algorithm).

$$\text{maximize } z = 7x_1 + 8x_2, \text{ subject to } \begin{cases} 4x_1 + x_2 \leq 100 \\ x_1 + x_2 \leq 80 \\ x_1 \leq 40 \\ x_1, x_2 \geq 0. \end{cases}$$

[**5**] Consider the linear program: Minimize $z = x_1 - x_2$ subject to

$$\begin{aligned} -x_1 + x_2 &\leq 1 \\ x_1 - 2x_2 &\leq 2 \\ x_1,\ x_2 &\geq 0. \end{aligned}$$

Derive an expression for the set of optimal solutions to this problem, and show that this set is unbounded.