Math 164: Homework #4, due on Wednesday, April 29, 2009

- Review Sections 4.1-4.3 from the textbook and solve additional problems.

[**1**] Solve the following linear program graphically:

$$\text{maximize } z = 6x_1 - 3x_2, \text{ subject to } \begin{cases} 2x_1 + 5x_2 \geq 10, \\ 3x_1 + 2x_2 \leq 40, \\ x_1, x_2 \leq 15. \end{cases}$$

[**2**] Convert the following linear program to standard form:

$$\text{minimize } z = x_1 - 5x_2 - 7x_3, \;\; \text{subject to } \begin{cases} 5x_1 - 2x_2 + 6x_3 \geq 5 \\ 3x_1 + 4x_2 - 9x_3 = 3 \\ 7x_1 + 3x_2 + 5x_3 \leq 9 \\ x_1 \geq -2, \;\; x_2, \;\; x_3 \text{ free.} \end{cases}$$

[**3**] Consider the system of linear constraints

$$\begin{array}{ccccc} 2x_1 & + & x_2 & \leq & 100, \\ x_1 & + & x_2 & \leq & 80, \\ x_1 & & & \leq & 40, \\ x_1 & , & x_2 & \geq & 0. \end{array}$$

(a) Write this system in standard form, and determine all the basic solutions (feasible and infeasible).

(b) Determine the extreme points of the feasible region (corresponding to both the standard form, as well as the original version).

[**4**] Consider a linear program with the constraints in standard form:

$$\{Ax = b, \;\; x \geq 0\}.$$

Prove that if $d \neq 0$ satisfies $Ad = 0$ and $d \geq 0$, then d is a direction of unboundedness.

[**5**] Consider the linear program: minimize $z = -5x_1 - 7x_2$, subject to

$$\begin{cases} -3x_1 + 2x_2 \leq 30 \\ -2x_1 + x_2 \leq 12 \\ x_1, x_2 \geq 0. \end{cases}$$

(a) Draw a graph of the feasible region

(b) Determine the extreme points of the feasible region

(c) Determine two linearly-independent directions of unboundedness.

(b) Convert the linear program to standard form and determine two linearly-independent directions of unboundedness for this version of the problem. Verify that the directions of unboundedness satisfy $Ad = \vec{0}$ and $d \geq \vec{0}$.