Math 155: Homework # 9, due on Friday, March 15

[**1**] Consider the motion blur in the frequency domain given by

$$H(u,v) = \int_0^T e^{-2\pi i[ux_0(t)+vy_0(t)]}dt.$$

For uniform motion given by $x_0(t) = \frac{at}{T}$ and $y_0(t) = \frac{bt}{T}$ (T=exposure time), show that the degradation function becomes

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin\left[\pi(ua+vb)\right]e^{-\pi i(ua+vb)}.$$

[**2**] **Parametric Wiener Filter**

(a) Implement a motion blurring filter as in problem [1].

(b) Blur image 5.26(a) in the +45o direction using $T = 1$, as in Fig. 5.26(b) ($a = b = 0.1$).

(c) Add a small amount of Gaussian noise of 0 mean to the blurred image. For this, you can use the Matlab command:

J = imnoise(I,'gaussian',0,v) adds Gaussian white noise of mean 0 and variance v to the image I (the parameter v needs to be specified). The default is zero mean noise with 0.01 variance.

(d) Restore the image using the parametric Wiener filter given by

$$\hat{F}(u,v) = \Big[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\Big]G(u,v),$$

where K is a specified constant, chosen to obtain best visual results.

(note that you may have to avoid division by zero in the above formula).

[**3**] Recall the 1D Laplacian of Gaussian (LoG) operator

$$\nabla^2 h(r) = \Big[\frac{r^2-\sigma^2}{\sigma^4}\Big]e^{-\frac{r^2}{2\sigma^2}}.$$

(a) Show that the integral value of the LoG operator $\nabla^2 h$ is zero. Hint: use the identities $\frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} e^{-r^2/(2\sigma^2)}dr = 1$, $\sigma^2 = \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} r^2 e^{-r^2/(2\sigma^2)}dr$.

(b) Prove that the integral value of any function convolved with this operator is also zero. Hint: use the frequency domain.

[**4**] (a) Consider the radial function $h_0(r) = e^{-r^2/2\sigma^2}$, where $r^2 = x^2+y^2$. Compute the 2D Laplacian $\nabla^2[h_0(\sqrt{x^2+y^2})]$.

(b) A certain X-ray imaging geometry produces a blurring degradation that can be modeled as the convolution of the sensed image with spatial, circularly symmetric function

$$h(r) = \Big[(r^2-2\sigma^2)/\sigma^4\Big]e^{-r^2/2\sigma^2},$$

where $r^2 = x^2 + y^2$. Show that the degradation in the frequency domain is given by the expression

$$H(u,v) = -8\pi^3\sigma^2(u^2+v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}.$$

- Optional Computational Question

[**5**] (a) Give the main steps of edge finding using the zero-crossings of the Laplacian of Gaussian.

(b) Implement and apply this method to the angiogram image from Fig. 10.15(a) (as in Fig. 10.22 for the house image, but you will apply your method to the angiogram image).