Math 155: Hw # 7, due on Friday, March 1st

[**1**] We have seen in continuous variables that $\mathcal{F}(\delta) \equiv 1$, thus δ and 1 form a Fourier pair; we also have that $\mathcal{F}(1) = \delta$. Using the second property and the translation property, show that the Fourier transform of $f(x) = \sin(2\pi u_0 x)$, where u_0 is a real number, is $F(u) = (i/2)\Big[\delta(u + u_0) - \delta(u - u_0)\Big]$.
(hint: you could express f function of exponentials).

[**2**] We have seen in continuous variables that $\mathcal{F}(\delta) \equiv 1$, thus δ and 1 form a Fourier pair; we also have that $\mathcal{F}(1) = \delta$. Using the second property and the translation property, show that the Fourier transform of the continuous function $f(x, y) = A\sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u, v) = A\frac{i}{2}\Big[\delta(u + u_0, v + v_0) - \delta(u - u_0, v - v_0)\Big].$$

(hint: you could express f function of exponentials as done in the 1D case).

[**3**] Assume that $\mathcal{F}(1) = \delta$ also holds in the discrete case (this can be shown). Using this property and the translation property, show that the Fourier transform of the discrete function $f(x, y) = \sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u, v) = \frac{i}{2}\Big[\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)\Big].$$

[**4**] **Periodic Noise Reduction Using a Notch Filter**

(a) Write a program that implements sinusoidal noise of the form: $n(x, y) = A\sin(2\pi u_0 x + 2\pi v_0 y)$.
The input to the program must be the amplitude, A, and the two frequency components u_0 and v_0.

(b) Download image 5.26(a) of size $M \times N$ and add sinusoidal noise to it, with $v_0 = 0$. The value of A must be high enough for the noise to be quite visible in the image (for example, you can take $A = 100$, $u_0 = 134.4$, $v_0 = 0$).

(c) Compute and display the degraded image and its spectrum (you may need to apply a log transform to visualize the spectrum).

(d) Notch-filter the image using a notch filter of the form shown in Fig. 5.19(c) to remove the periodic noise.

[**5**] (a) Recall the definition of the convolution $f * g(x, y)$ in continuous variables in two dimensions.

(b) Show that $\nabla^2(f * g) = f * (\nabla^2 g)$, where ∇^2 denotes the Laplace operator in (x, y).