Math 155. Instructor: Luminita Vese.
Teaching Assistant: Siting Liu.

Homework # 6, due on Friday, February 22

[**1**] (a) Show in discrete variables that

$$\mathcal{F}\Big(f(x,y)e^{2\pi i(u_0\frac{x}{M}+v_0\frac{y}{N})}\Big) = F(u-u_0, v-v_0),$$

where $F = \mathcal{F}(f)$.

(b) Using (a), deduce the formula used in shifting the center of the transform by multiplication with $(-1)^{x+y}$, when $u_0 = M/2$ and $v_0 = N/2$, with M and N even positive integers.

[**2**]

(a) Show, in discrete variables, the translation property

$$\mathcal{F}\Big(f(x-x_0, y-y_0)\Big) = F(u,v)e^{-2\pi i(x_0u/M+y_0v/N)},$$

where $F(u,v) = \mathcal{F}(f(x,y))$.

(b) Consider the linear difference operator $g(x,y) = f(x+1,y) - f(x,y)$. Obtain the filter transfer function, $H(u,v)$, for performing the equivalent process in the frequency domain.

[**3**] Prove the validity of the discrete convolution theorem in one variable (you may need to use the translation properties).

[**4**] Assume that $f(x)$ is given by the discrete IFT formula in one dimension. Show the periodicity property $f(x) = f(x+kM)$, where k is an integer.

[**5**] (a) Implement the Gaussian lowpass filter in Eq. (4.3-8), using a radius $D_0 = 25$, and apply the algorithm to Fig4.11(a).

(b) Highpass the input image used in (a), using a highpass Gaussian filter of radius $D_0 = 25$ (see eq. (4.4-4)).

[**6**] Consider the two-dimensional continuous case.
(a) Write the Inverse Fourier transform formula by expressing $f(x,y)$ function of $F(u,v)$.
(b) Assume f is twice differentiable. Using (a), find the Fourier transform of the mixed partial derivative $\frac{\partial^2 f}{\partial x \partial y}$, function of F.