Math 155. Instructor: Luminita Vese.
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Homework # 6, due on Friday, February 22

[1] (a) Show in discrete variables that
\[ F(f(x, y) e^{2\pi i (u_0 x + v_0 y)}) = F(u - u_0, v - v_0), \]
where \( F = F(f) \).

(b) Using (a), deduce the formula used in shifting the center of the transform by multiplication with \((-1)^{x+y}\), when \( u_0 = M/2 \) and \( v_0 = N/2 \), with \( M \) and \( N \) even positive integers.

[2] (a) Show, in discrete variables, the translation property
\[ F(f(x-x_0, y-y_0)) = F(u, v) e^{-2\pi i (x_0 u/M + y_0 v/N)}, \]
where \( F(u, v) = F(f(x, y)) \).

(b) Consider the linear difference operator \( g(x, y) = f(x+1, y) - f(x, y) \). Obtain the filter transfer function, \( H(u, v) \), for performing the equivalent process in the frequency domain.

[3] Prove the validity of the discrete convolution theorem in one variable (you may need to use the translation properties).

[4] Assume that \( f(x) \) is given by the discrete IFT formula in one dimension. Show the periodicity property \( f(x) = f(x + kM) \), where \( k \) is an integer.

[5] (a) Implement the Gaussian lowpass filter in Eq. (4.3-8), using a radius \( D_0 = 25 \), and apply the algorithm to Fig4.11(a).

(b) Highpass the input image used in (a), using a highpass Gaussian filter of radius \( D_0 = 25 \) (see eq. (4.4-4)).

(a) Write the Inverse Fourier transform formula by expressing \( f(x, y) \) function of \( F(u, v) \).

(b) Assume \( f \) is twice differentiable. Using (a), find the Fourier transform of the mixed partial derivative \( \frac{\partial^2 f}{\partial x \partial y} \), function of \( F \).