Math 155: Hw # 7, due on Friday, March 2nd

[**1**] Consider the two-dimensional continuous case.
(a) Write the Inverse Fourier transform formula by expressing $f(x, y)$ function of $F(u, v)$.
(b) Assume f is twice differentiable. Using (a), find the Fourier transform of the mixed partial derivative $\frac{\partial^2 f}{\partial x \partial y}$, function of F.

[**2**] We have seen in continuous variables that $\mathcal{F}(\delta) \equiv 1$, thus δ and 1 form a Fourier pair; we also have that $\mathcal{F}(1) = \delta$. Using the second property and the translation property, show that the Fourier transform of the continuous function $f(x, y) = A\sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u, v) = A\frac{i}{2}\Big[\delta(u + u_0, v + v_0) - \delta(u - u_0, v - v_0)\Big].$$

(hint: you could express f function of exponentials as done in class in the 1D case).

[**3**] Assume that $\mathcal{F}(1) = \delta$ also holds in the discrete case (this can be shown). Using this property and the translation property, show that the Fourier transform of the discrete function $f(x, y) = \sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u, v) = \frac{i}{2}\Big[\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)\Big].$$

[**4**] **Periodic Noise Reduction Using a Notch Filter**

(a) Write a program that implements sinusoidal noise of the form:
$n(x, y) = A\sin(2\pi u_0 x + 2\pi v_0 y)$.
The input to the program must be the amplitude, A, and the two frequency components u_0 and v_0.

(b) Download image 5.26(a) of size $M \times N$ and add sinusoidal noise to it, with $v_0 = 0$. The value of A must be high enough for the noise to be quite visible in the image (for example, you can take $A = 100$, $u_0 = 134.4$, $v_0 = 0$).

(c) Compute and display the degraded image and its spectrum (you may need to apply a log transform to visualize the spectrum).

(d) Notch-filter the image using a notch filter of the form shown in Fig. 5.19(c), to remove the periodic noise.