

**Math 155. Instructor: Luminita Vese.
Teaching Assistant: Jacob Moorman.**

Homework # 6, due on Friday, February 23

[1] (a) Show in discrete variables that

$$\mathcal{F}\left(f(x, y)e^{2\pi i(u_0 \frac{x}{M} + v_0 \frac{y}{N})}\right) = F(u - u_0, v - v_0),$$

where $F = \mathcal{F}(f)$.

(b) Using (a), deduce the formula used in shifting the center of the transform by multiplication with $(-1)^{x+y}$, when $u_0 = M/2$ and $v_0 = N/2$, with M and N even positive integers.

[2]

(a) Show, in discrete variables, the translation property

$$\mathcal{F}\left(f(x - x_0, y - y_0)\right) = F(u, v)e^{-2\pi i(x_0 u/M + y_0 v/N)},$$

where $F(u, v) = \mathcal{F}(f(x, y))$.

(b) Consider the linear difference operator $g(x, y) = f(x + 1, y) - f(x, y)$. Obtain the filter transfer function, $H(u, v)$, for performing the equivalent process in the frequency domain.

[3] Prove the validity of the discrete convolution theorem in one variable (you may need to use the translation properties).

[4] Assume that $f(x)$ is given by the discrete IFT formula in one dimension. Show the periodicity property $f(x) = f(x + kM)$, where k is an integer.

[5] (a) Implement the Gaussian lowpass filter in Eq. (4.3-8), using a radius $D_0 = 25$, and apply the algorithm to Fig4.11(a).

(b) Highpass the input image used in (a), using a highpass Gaussian filter of radius $D_0 = 25$ (see eq. (4.4-4)).