Math 155. Instructor: Luminita Vese. Teaching Assistant: Jacob Moorman.

Homework # 3 Due on Friday, February 2

[**1**] *Computational project: Spatial filtering*
Consider the noisy X-ray image of circuit board corrupted by salt-and-pepper noise. Filter this image by applying a linear average filter with a 3×3 mask (use the average mask with entries $w_{s,t} = \frac{1}{9}$, for all $s, t \in \{-1, 0, 1\}$). You can keep the border pixels unchanged.

[**2**] The median, ξ, of a set of numbers is such that half the values in the set are less than or equal to ξ, and half are greater than or equal to ξ. For example, the median of the set of values $\{2, 3, 8, 20, 21, 25, 31\}$ is 20. Show that an operator applied to the set of images (matrices) of the same dimension, that computes the median, is nonlinear.

[**3**] Write a computer program that will denoise an image using the 3x3 median filter. Apply your algorithm to the X-Ray image of circuit board corrupted by salt-and-pepper noise (Fig3.37(a).jpg). You should turn in the details of the method, your computer program, the input and output images. For simplicity, perform your calculations only for interior pixels, not for boundary pixels. Explain your result and compare it with the output obtained using the linear average filter.

[**4**] Show that the continuous Laplacian is a linear operation, in other words show that the mapping $f \mapsto \nabla^2 f$ is linear on the vector space of functions $f \in C^2$ in two dimensions (continuous and twice differentiable functions).