Math 155: Instructor: Luminita Vese. Teaching Assistant: Jacob Moorman.

Homework # 2   Due on Wednesday January 24

[**1**] Give a single intensity transformation function T for spreading the intensities of an image so the lowest intensity is 0 and the highest is $L-1$.

[**2**] (Histogram equalization in continuous variables) An image has the gray-level PDF

$$p_r(r) = \begin{cases} \frac{6r+2}{3(L-1)^2+2(L-1)} \text{ if } 0 \le r \le L-1 \\ 0, \text{ otherwise} \end{cases}$$

with $L-1>0$.

(a) Verify some of the properties that a PDF has to satisfy: $p_r(r) \ge 0$ for all $r \in (-\infty, \infty)$ and $\int_{-\infty}^{\infty} p_r(r)dr = 1$.

(b) Find the tranformation function $s = T(r)$ obtained through "histogram equalization" in continuous variables.

(c) Verify that $p_s(s)$ is a uniform "flat" distribution for $s \in [0,1]$ (recall the formula $p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$).

[**3**] (Histogram matching in continuous variables) An image has the gray-level PDF $p_r(r) = -2r+2$, with $0 \le r \le 1$. It is desired to transform the gray levels of this image so that they will have the specified $p_z(z) = 2z$, $0 \le z \le 1$. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this (here $L-1=1$).

[**4**] A linear spatial filter of size $(2a+1)\times(2b+1)$ defined by the transformation H, $g = H[f]$, is given by

$$g(x,y) = \sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s, y+t),$$

where $f(x,y)$ is a given input image, a, b are positive integers, and $w(s,t)$ are weights for $-a \le s \le a$, $-b \le t \le b$.

(a) Give the definition of a linear transformation $H: V \to V$, where V is a vector space.

(b) Show that H defined above is indeed a linear transformation (assume images defined on the entire plane, or ignore border effects).