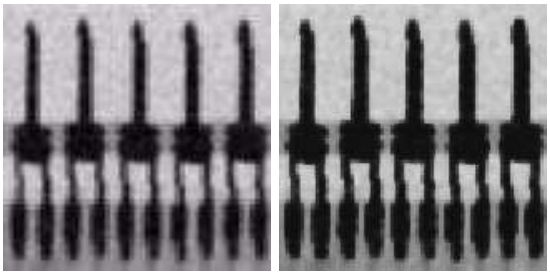

Math 155: Homework # 8, due on Friday, March 10, or the following Monday

[1] The two subimages shown were extracted from the top, right corners of Figs. 5.7(c) and (d), respectively. Thus, the subimage on the left is the result of using an arithmetic mean filter of size 3x3; the other subimage is the result of using a geometric mean filter of the same size.

(a) Explain why the subimage obtained with geometric mean filtering is less blurred. *Hint:* Start your analysis by examining a 1-D step edge profile.

(b) Explain why the black components in the right image are thicker.

[2] Download from the class web page the image Fig5.07(b).jpg (X-Ray image corrupted by Gaussian noise).

(a) Write a computer program to implement the arithmetic mean filter of size 3x3. Apply the program to the image Fig5.07(b).jpg

(b) Write a computer program to implement the geometric mean filter of size 3x3. Apply the program to the image Fig5.07(b).jpg

(c) Explain your results. Evaluate the SNR (signal-to-noise-ratio) for both results in (a) and (b) (before denoising and after denoising). Note, higher SNR, better denoised image. Let $\hat{f}$ be the denoised image, and f the clean true image. Then $SNR = 10 \log_{10} \frac{\sum_{x,y} (\hat{f})^2}{\sum_{x,y} (f - \hat{f})^2}$. To evaluate the SNR before denoising, substitute $\hat{f}$ by g in the above formula.

[3] Refer to the contraharmonic filter given in Eq. (5.3-6).

(a) Explain why the filter is effective in eliminating pepper noise when Q is positive.

(b) Explain why the filter is effective in eliminating salt noise when Q is negative.

(c) Explain why the filter gives poor results (such as the results shown in Fig. 5.9) when the wrong polarity is chosen for Q.

(d) Discuss the behavior of the filter when $Q = -1$.

(e) Discuss (for positive and negative Q) the behavior of the filter in areas of constant gray levels.

[4] (a) Download from the class web page the image Fig5.08(a).jpg (X-Ray image corrupted by pepper noise). Write a computer program that will filter this image with a 3x3 contraharmonic filter of order 1.5.

(b) Download from the class web page the image Fig5.08(b).jpg (X-Ray image corrupted by salt noise). Write a computer program that will filter this image with a 3x3 contraharmonic filter of order -1.5.

[5] (a) Recall the definition of the convolution $f * g(x, y)$ in continuous variables and two dimensions.

(b) Show that $\nabla^2(f * g) = f * (\nabla^2 g)$, where ∇^2 denotes the Laplace operator in (x, y).