Math 155. Instructor: Luminita Vese. Teaching Assistant: Baichuan Yuan.

Homework # 3 Due on Friday, February 3

[**1**] *Computational project: Spatial filtering* Consider the noisy X-ray image of circuit board corrupted by salt-and-pepper noise. Filter this image by applying a linear average filter with a 3×3 mask (use the average mask with entries $w_{s,t} = \frac{1}{9}$, for all $s, t \in \{-1, 0, 1\}$). You can keep the border pixels unchanged.

[**2**] Show that the Laplacian operation $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ is isotropic (invariant under rotations, or rotationally invariant). You will need the following equations relating coordinates after axis rotation by an angle θ:

$$x = x' \cos\theta - y' \sin\theta$$

$$y = x' \sin\theta + y' \cos\theta$$

where (x, y) are the unrotated and (x', y') are the rotated coordinates.

[**3**] Show that the continuous Laplacian is a linear operation, in other words show that the mapping $f \mapsto \nabla^2 f$ is linear on the vector space of functions $f \in C^2$ in two dimensions (continuous and twice differentiable functions).

[**4**]

(a) Show that the magnitude of the gradient $|\nabla f| = \sqrt{(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}$ is an isotropic operation.

(b) Show that the isotropic property is lost in general if the gradient magnitude is approximated by $|\nabla f| \approx |\frac{\partial f}{\partial x}| + |\frac{\partial f}{\partial y}|$.