Homework # 3 Due on Friday, February 3

[1] **Computational project: Spatial filtering** Consider the noisy X-ray image of circuit board corrupted by salt-and-pepper noise. Filter this image by applying a linear average filter with a $3 \times 3$ mask (use the average mask with entries $w_{s,t} = \frac{1}{9}$, for all $s, t \in \{-1, 0, 1\}$). You can keep the border pixels unchanged.

[2] Show that the Laplacian operation $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ is isotropic (invariant under rotations, or rotationally invariant). You will need the following equations relating coordinates after axis rotation by an angle $\theta$:

\[
\begin{align*}
x &= x' \cos \theta - y' \sin \theta \\
y &= x' \sin \theta + y' \cos \theta
\end{align*}
\]

where $(x, y)$ are the unrotated and $(x', y')$ are the rotated coordinates.

[3] Show that the continuous Laplacian is a linear operation, in other words show that the mapping $f \mapsto \nabla^2 f$ is linear on the vector space of functions $f \in C^2$ in two dimensions (continuous and twice differentiable functions).

[4]

(a) Show that the magnitude of the gradient $|\nabla f| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$ is an isotropic operation.

(b) Show that the isotropic property is lost in general if the gradient magnitude is approximated by $|\nabla f| \approx |\frac{\partial f}{\partial x}| + |\frac{\partial f}{\partial y}|$. 