Problems #3

[**1**] The median, ξ, of a set of numbers is such that half the values in the set are less than or equal to ξ, and half are greater than or equal to ξ. For example, the median of the set of values $\{2, 3, 8, 20, 21, 25, 31\}$ is 20. Show that an operator applied to the set of images (matrices) of the same dimension, that computes the median, is nonlinear.

[**2**] Refer to the contraharmonic filter given in the lecture notes.
(a) Explain why the filter is effective in eliminating pepper noise when Q is positive.
(b) Explain why the filter is effective in eliminating salt noise when Q is negative.
(c) Explain why the filter gives poor results when the wrong polarity is chosen for Q.
(d) Discuss the behavior of the filter when $Q = -1$.
(e) Discuss (for positive and negative Q) the behavior of the filter in areas of constant gray levels.

[**3**] We have seen in continuous variables that $\mathcal{F}(\delta) \equiv 1$, thus δ and 1 form a Fourier pair; we also have that $\mathcal{F}(1) = \delta$. Using the second property and the translation property, show in one dimension that the Fourier transform of $f(x) = \sin(2\pi u_0 x)$, where u_0 is a real number, is $F(u) = (i/2)\Big[\delta(u + u_0) - \delta(u - u_0)\Big]$.
(hint: you could express f function of exponentials).

[**4**] We have seen in continuous variables that $\mathcal{F}(\delta) \equiv 1$, thus δ and 1 form a Fourier pair; we also have that $\mathcal{F}(1) = \delta$. Using the second property and the translation property, show that the Fourier transform of the continuous function $f(x, y) = A\sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u, v) = A\frac{i}{2}\Big[\delta(u + u_0, v + v_0) - \delta(u - u_0, v - v_0)\Big].$$

(hint: you could express f function of exponentials).

[**5**] Assume that $\mathcal{F}(1) = \delta$ also holds in the discrete case (this can be shown). Using this property and the translation property, show that the Fourier transform of the discrete function $f(x, y) = \sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u, v) = \frac{i}{2}\Big[\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)\Big].$$

[**6**] Consider the motion blur in the frequency domain given by

$$H(u, v) = \int_0^T e^{-2\pi i[ux_0(t) + vy_0(t)]} dt.$$

For uniform motion given by $x_0(t) = \frac{at}{T}$ and $y_0(t) = \frac{bt}{T}$ (T=exposure time), show that the degradation function becomes

$$H(u, v) = \frac{T}{\pi(ua + vb)} \sin\left[\pi(ua + vb)\right] e^{-\pi i(ua + vb)}.$$