

An Introduction to Mathematical Image Processing

IAS, Park City Mathematics Institute, Utah
Undergraduate Summer School 2010

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Lecture Meeting Time: Mon, Tue, Thu, Fri 1-2pm (location: Coalition 1&2)
(if preferable, we can schedule a 5th lecture every week ?)

Problem Session: 4.30-5.30, 5.30-6.30 (location: computer lab or tent)

Course Web Page:

http://www.math.ucla.edu/~lvese/155.1.09w/PCMI_USS_2010

Lectures, Problems and Computer Projects are posted on the course webpage.

Link to PCMI Forum Discussion (ning): for messages, comments, updates,
course material <http://parkcitymathematicsinstitute.ning.com>

Introductory tutorial on *Image Processing Using Matlab* (by Pascal Getreuer):
<http://www.math.ucla.edu/~getreuer/matlabimaging.html>

For an introduction to image processing, a useful reading textbook is:

R.C. Gonzalez and R.E. Woods, Digital Image Processing, 3rd edition, Prentice-Hall.

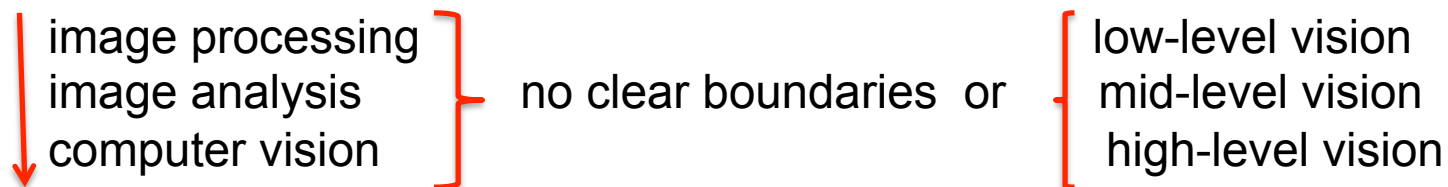
See also

R.C. Gonzalez, R.E. Woods and S.L. Eddins, Digital Image Processing Using Matlab, 2nd edition, Prentice-Hall.

Please visit also the webpage of these two textbooks for review material, computational projects, problem solutions, images used in the textbook, imaging toolbox, and many other useful information:

<http://www.imageprocessingplace.com/index.htm>

Areas:



Fundamental Steps in Image Processing and Computer Vision

Image acquisition: output = digital image

Lets represent a continuous image as a 2D function $f(x,y)$ which can be characterized by two components:

- (1) the amount of source illumination incident on the scene (illumination) $i(x,y)$
- (2) the amount of illumination reflected by the objects (reflectance) $r(x,y)$

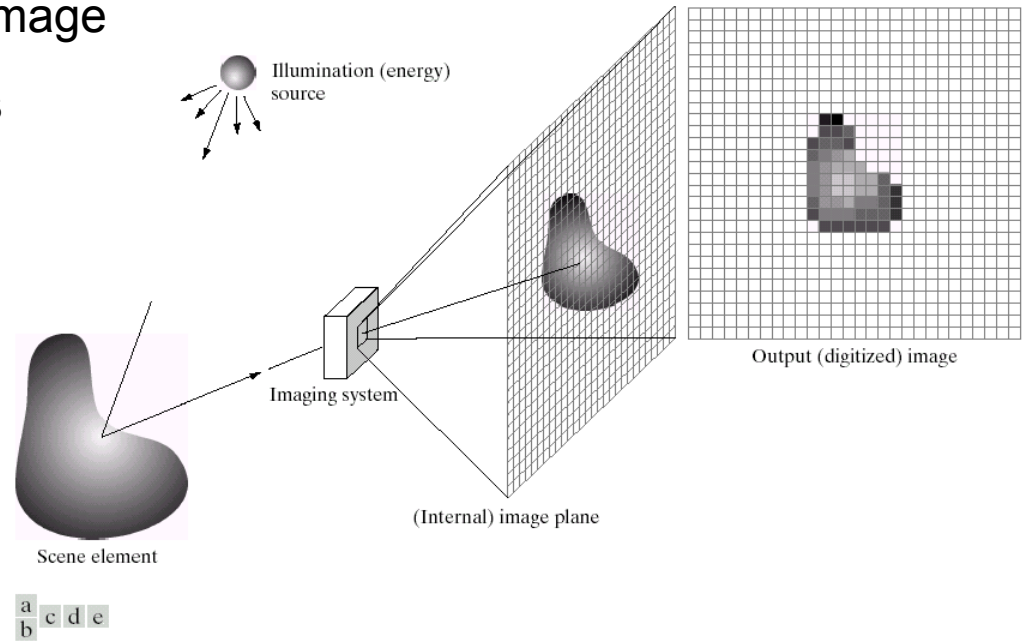


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy (“illumination”) source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

We have $f(x,y)=i(x,y)r(x,y)$ where $0<i(x,y)<\infty$ and $0<r(x,y)<1$

Reflectance is bounded below by 0 (total absorption) and above by 1 (total reflectance)

$i(x,y)$ depends on the illumination source

$r(x,y)$ depends on the characteristics of the imaged objects.

The same expressions are applicable to images formed via transmission of the illumination through a medium (chest X-ray, etc). We deal with *transmissivity* instead of *reflectivity* (image = absorption coefficient of the tissue)

We have:

$L_{\min} \leq f(x,y) \leq L_{\max}$ where $l = f(x,y)$ is the gray - level at coordinates (x,y)

It is common to shift the interval $[L_{\min}, L_{\max}]$ to the interval $[0, L - 1]$

Then $l = 0$ is considered black and $l = L - 1$ is considered white on the gray scale

The intermediate values are shades varying from black to white.

Image sampling and quantization

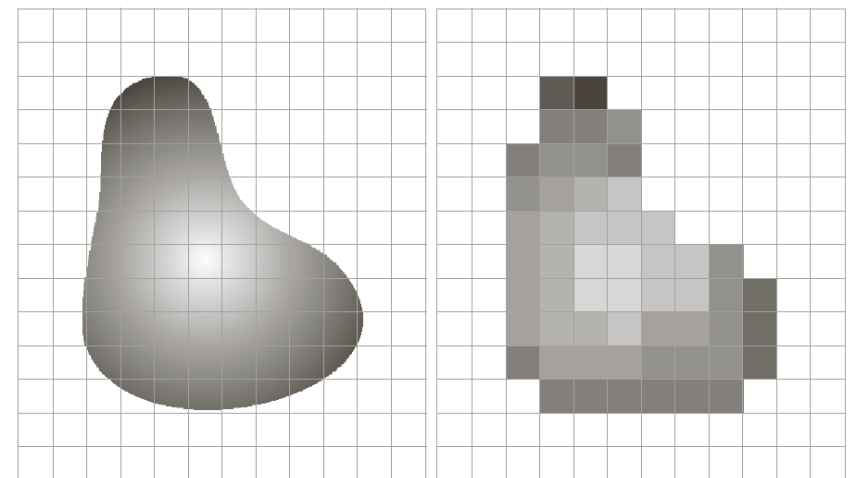
Need to convert the continuous sensed data into digital form via two processes:
sampling and *quantization*.

Sampling = digitizing the coordinate values

Quantization = digitizing the amplitude values

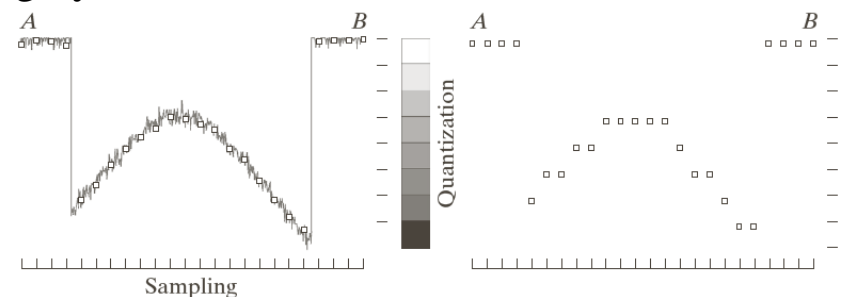
Digital image: $L = 2^k$ gray levels (e.g., 256 integer gray levels 0, 1,...,255)

$$f = \begin{bmatrix} f_{0,0} & f_{0,1} & \cdots & f_{0,N-1} \\ f_{1,0} & f_{1,1} & \cdots & f_{1,N-1} \\ \vdots & \vdots & \vdots & \vdots \\ f_{M-1,0} & f_{M-1,1} & \cdots & f_{M-1,N-1} \end{bmatrix} = M \times N \text{ matrix}$$



a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.



Low-level vision: input = image, output = image

Image enhancement: subjective process (e.g. image sharpening)

Image Restoration: objective process, denoising, deblurring, inpainting (depends on the degradation)



Color Image Processing: several color modes; color (R,G,B) images are represented by a vector-valued function of three components; natural extensions from gray-scale to color images (most of the time)

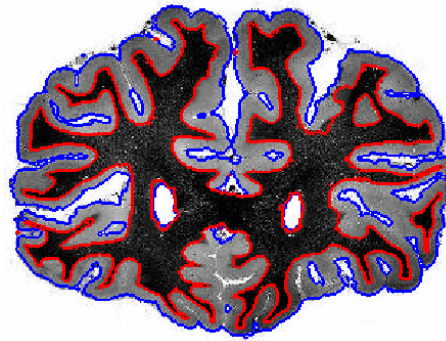
Wavelets: advance course, mathematical tool to allow representation of images at various degrees of resolutions, used in many image processing tasks

Compression: reducing the storage required to save an image (jpeg 2000)

Midd-level vision: input = image, output = image attributes

Morphological Processing: tools for extracting image components useful in the representation and description of shape.

Segmentation: partition an image into its constituent parts or objects



High-level vision: input and output = image attributes

Representation and description: following segmentation, gives representation of boundaries and regions; description given by a set of object attributes or features.

Recognition: assigning a label (e.g., ``vehicle") to an object based on its descriptors or features

This course will deal with low-level and mid-level tasks.

Topics to be covered

Introduction (fundamental steps in image processing and computer vision, continuous and digital images)

A simple image formation model

Image sampling and quantization

Intensity transformations and spatial filtering

- Histogram equalization
- Spatial filtering (correlation, convolution, spatial filter masks)
- Smoothing spatial filters
- Sharpening spatial filters using the Laplacian

Filtering in the frequency domain

- 1D and 2D continuous and discrete Fourier transforms
- convolution theorem
- properties of the Fourier transform
- filtering in the frequency domain (smoothing and sharpening, low-pass and high-pass filtering)
- the Laplacian in the frequency domain, enhancement
- homomorphic filtering
- band-reject and band-pass filters

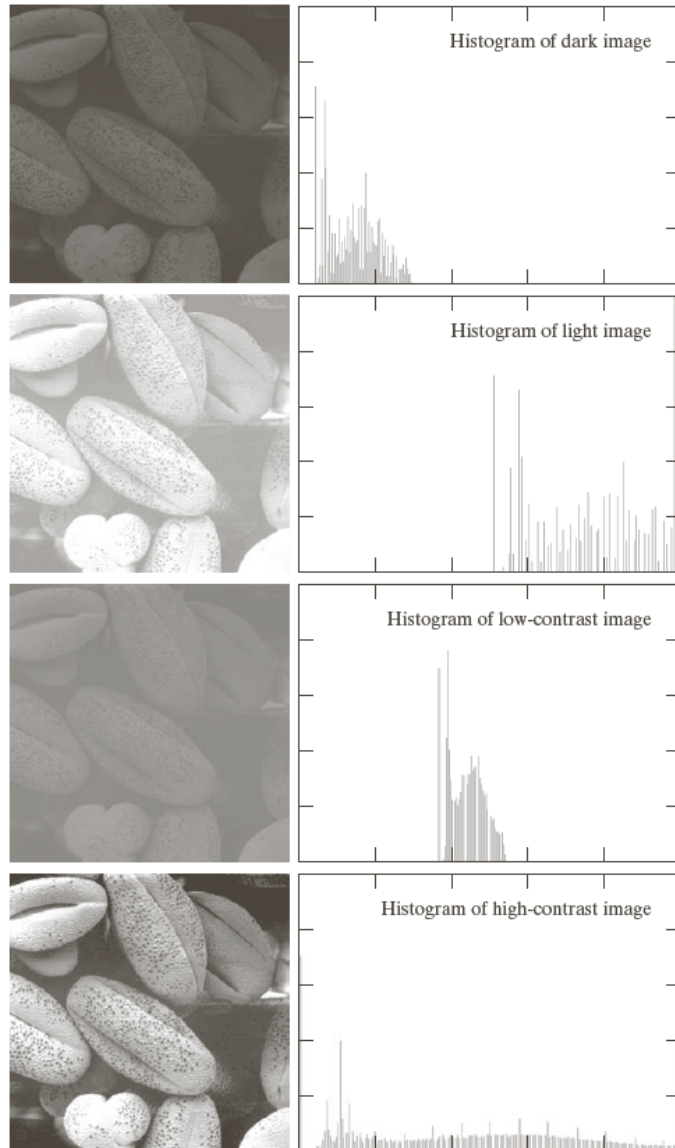
Image restoration and reconstruction

- noise models
- mean filters
- order statistics (nonlinear) filters
- adaptive median filter
- periodic noise reduction
- NL Means filter
- linear, position-invariant degradations
- examples of degradation (PSF) functions
- inverse filtering (Wiener filter, constrained least squares filtering, total variation minimization)
- image reconstruction from projections (Radon transform, computed tomography, the Fourier slice theorem, filtered backprojections using parallel beam)

Image segmentation

- image gradient, gradient operators, gradient-based edge detection
- the Marr-Hildreth edge detector, Canny edge detector
- active contours and curve evolution for object detection
- global processing using the Hough transform

Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms



Middle: histogram-equalized images. Right: corresponding histograms

