Math 155: Homework # 9, due on Friday, March 13 (6 questions)

[**1**] Consider a linear, position-invariant image degradation system with impulse response

$$h(x-\alpha, y-\beta) = e^{-[(x-\alpha)^2+(y-\beta)^2]}.$$

Suppose that the input to the system is an image consisting of a line of infinitesimal width located at $x = a$, and modeled by $f(x, y) = \delta(x - a)$, where δ is the impulse function. Assuming no noise, what is the output image $g(x, y)$?

Hint: Use equation (5.5-13) and also that

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}\ dz = 1,$$

where μ and σ are constants.

[**2**] (a) Consider the radial function $h_0(r) = e^{-r^2/2\sigma^2}$, where $r^2 = x^2 + y^2$. Compute the 2D Laplacian $\nabla^2[h_0(\sqrt{x^2+y^2})]$.

(b) A certain X-ray imaging geometry produces a blurring degradation that can be modeled as the convolution of the sensed image with spatial, circularly symmetric function

$$h(r) = \Big[(r^2 - 2\sigma^2)/\sigma^4\Big]e^{-r^2/2\sigma^2},$$

where $r^2 = x^2 + y^2$. Show that the degradation in the frequency domain is given by the expression

$$H(u, v) = -8\pi^3\sigma^2(u^2 + v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}.$$

[**3**] **Parametric Wiener Filter**

(a) Implement a motion blurring filter as in problem [6], Hw # 8.

(b) Blur image 5.26(a) in the +45o direction using $T = 1$, as in Fig. 5.26(b) ($a = b = 0.1$).

(c) Add a small amount of Gaussian noise of 0 mean to the blurred image. For this, you can use the Matlab command:

J = imnoise(I,'gaussian',0,v) adds Gaussian white noise of mean 0 and variance v to the image I (the parameter v needs to be specified). The default is zero mean noise with 0.01 variance.

(d) Restore the image using the parametric Wiener filter given by

$$\hat{F}(u, v) = \Big[\frac{1}{H(u, v)}\frac{|H(u, v)|^2}{|H(u, v)|^2 + K}\Big]G(u, v),$$

where K is a specified constant, chosen to obtain best visual results.

(note that you may have to avoid division by zero in the above formula).

[**4**] (a) Recall the definition of the convolution $f*g(x, y)$ in continuous variables and two dimensions.

(b) Show that $\nabla^2(f * g) = f * (\nabla^2 g)$, where ∇^2 denotes the Laplace operator in (x, y).

[**5**] (a) Read Section 10.2 (pages 692-717).

(b) Give the main steps of edge finding using the zero-crossings of the Laplacian of Gaussian (read pages 714-717).

(c) Implement and apply this method to the angiogram image from Fig. 10.15(a) (as in Fig. 10.22 for the house image, but you will apply your method to the angiogram image).

[6] Recall the 1D Laplacian of Gaussian (LoG) operator

$$\nabla^2 h(r) = \Big[\frac{r^2 - \sigma^2}{\sigma^4}\Big] e^{-\frac{r^2}{2\sigma^2}}.$$

(a) Show that the average value of the LoG operator $\nabla^2 h$ is zero. Hint: use the identities $\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} e^{-r^2/(2\sigma^2)} dr = 1,\ \sigma^2 = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{+\infty} r^2 e^{-r^2/(2\sigma^2)} dr.$

(b) Prove that the average value of any function convolved with this operator is also zero. Hint: use the frequency domain.