Math 155: Homework # 8, due on Friday, March 6

[1] Periodic Noise Reduction Using a Notch Filter

(a) Write a program that implements sinusoidal noise of the form given in the previous homework: $n(x, y) = A\sin(2\pi u_0 x + 2\pi v_0 y)$. The input to the program must be the amplitude, A, and the two frequency components u_0 and v_0.

(b) Download image 5.26(a) of size $M \times N$ and add sinusoidal noise to it, with $v_0 = 0$. The value of A must be high enough for the noise to be quite visible in the image (for example, you can take $A = 100$, $u_0 = 134.4$, $v_0 = 0$).

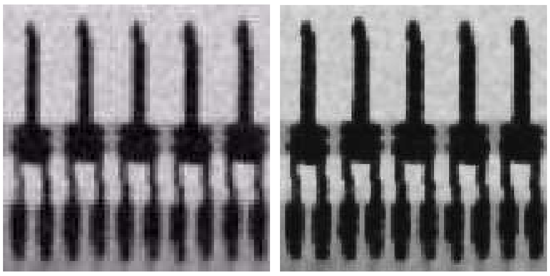
(c) Compute and display the degraded image and its spectrum (you may need to apply a log transform to visualize the spectrum).

(d) Notch-filter the image using a notch filter of the form shown in Fig. 5.19(c), to remove the periodic noise.

[**2**] The two subimages shown were extracted from the top, right corners of Figs. 5.7(c) and (d), respectively. Thus, the subimage on the left is the result of using an arithmetic mean filter of size 3x3; the other subimage is the result of using a geometric mean filter of the same size.

(a) Explain why the subimage obtained with geometric mean filtering is less blurred. *Hint:* Start your analysis by examining a 1-D step edge profile.

(b) Explain why the black components in the right image are thicker.



[**3**] Download from the class web page the image Fig5.07(b).jpg (X-Ray image corrupted by Gaussian noise).

(a) Write a computer program to implement the arithmetic mean filter of size 3x3. Apply the program to the image Fig5.07(b).jpg

(b) Write a computer program to implement the geometric mean filter of size 3x3. Apply the program to the image Fig5.07(b).jpg

(c) Explain your results. Evaluate the SNR (signal-to-noise-ratio) for both results in (a) and (b) (before denoising and after denoising). Note, higher SNR, better denoised image. Let $\hat{f}$ be the denoised image, and f the clean true image. Then $SNR = 10\log_{10}\frac{\sum_{x,y}(\hat{f})^2}{\sum_{x,y}(f-\hat{f})^2}$. To evaluate the SNR before denoising, substitute $\hat{f}$ by g in the above formula.

[4] Refer to the contraharmonic filter given in Eq. (5.3-6).

(a) Explain why the filter is effective in eliminating pepper noise when Q is positive.

(b) Explain why the filter is effective in eliminating salt noise when Q is negative.

(c) Explain why the filter gives poor results (such as the results shown in Fig. 5.9) when the wrong polarity is chosen for Q.

(d) Discuss the behavior of the filter when $Q = -1$.

(e) Discuss (for positive and negative Q) the behavior of the filter in areas of constant gray levels.

[5] (a) Download from the class web page the image Fig5.08(a).jpg (X-Ray image corrupted by pepper noise). Write a computer program that will filter this image with a 3x3 contraharmonic filter of order 1.5.

(b) Download from the class web page the image Fig5.08(b).jpg (X-Ray image corrupted by salt noise). Write a computer program that will filter this image with a 3x3 contraharmonic filter of order -1.5.

[6] Consider the motion blur in the frequency domain given by

$$H(u, v) = \int_0^T e^{-2\pi i[ux_0(t)+vy_0(t)]} dt.$$

For uniform motion given by $x_0(t) = \frac{at}{T}$ and $y_0(t) = \frac{bt}{T}$ (T=exposure time), show that the degradation function becomes

$$H(u, v) = \frac{T}{\pi(ua + vb)} \sin\left[\pi(ua + vb)\right] e^{-\pi i(ua+vb)}.$$