Math 155: Hw # 7, due on Friday, Feb. 27 or Monday, March 2

[**1**] Consider the two-dimensional continuous case.
(a) Recall the Inverse Fourier transform formula in 2D (express $f(x,y)$ function of $F(u,v)$).
(b) Assume f is twice differentiable. Using (a), find the Fourier transform of the mixed partial derivative $\frac{\partial^2 f}{\partial x \partial y}(x,y)$, function of F.

[**2**] We have seen in continuous variables that $\mathcal{F}(\delta) \equiv 1$, thus δ and 1 form a Fourier pair; we also have that $\mathcal{F}(1) = \delta$. Using the second property and the translation property, show that the Fourier transform of $f(x) = \sin(2\pi u_0 x)$, where u_0 is a real number, is $F(u) = (i/2)\Big[\delta(u+u_0) - \delta(u-u_0)\Big]$.
(hint: you could express f function of exponentials).

[**3**] We have seen in continuous variables that $\mathcal{F}(\delta) \equiv 1$, thus δ and 1 form a Fourier pair; we also have that $\mathcal{F}(1) = \delta$. Using the second property and the translation property, show that the Fourier transform of the continuous function $f(x,y) = A\sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u,v) = A\frac{i}{2}\Big[\delta(u+u_0, v+v_0) - \delta(u-u_0, v-v_0)\Big].$$

(hint: you could express f function of exponentials).

[**4**] Assume that $\mathcal{F}(1) = \delta$ also holds in the discrete case (this can be shown). Using this property and the translation property, show that the Fourier transform of the discrete function $f(x,y) = \sin(2\pi u_0 x + 2\pi v_0 y)$ is

$$F(u,v) = \frac{i}{2}\Big[\delta(u+Mu_0, v+Nv_0) - \delta(u-Mu_0, v-Nv_0)\Big].$$

Note: applications of exercises 2-4 will be seen in more details in chapter 5, for removing periodic noise, such as sinusoidal noise.