Math 155: Homework # 5, due on Friday, February 13

[**1**] Recall that the 1D DFT is

$$F(u) = \sum_{x=0}^{M-1} f(x) e^{-2\pi i u x/M}.$$

(a) Assuming the formula above, show the identity

$$f(x) = \frac{1}{M} \sum_{u=0}^{M-1} F(u) e^{2\pi i u x/M},$$

using the following orthogonality of exponentials

$$\sum_{u=0}^{M-1} e^{-2\pi i u y/M} e^{2\pi i u x/M} = \begin{cases} M & \text{if } x = y \\ 0 & \text{otherwise.} \end{cases}$$

(b) Show now the converse of (a): assume given $f(x)$ function of $F(u)$ in the discrete case, and show the identity for $F(u)$ (use the same orthogonality of exponentials).

[**2**] Show that the continuous 2D Fourier transform is a linear process.

[**3**] Compute in continuous variables the Fourier transform of the function

$$f(x) = \begin{cases} A, & \text{if } 0 \le x \le K, \\ 0, & \text{otherwise,} \end{cases}$$

where A and K are positive constants. Evaluate $F(0)$.

[**4**] Consider again the 2D continous Fourier transform and its inverse (denote by $H(u,v)$ the 2D Fourier transform of the spatial filter $h(x,y)$). Show that if the transform $H(u,v)$ is real and symmetric, i.e. if

$$H(u,v) = \overline{H(u,v)} = \overline{H(-u,-v)} = H(-u,-v),$$

then the corresponding spatial domain filter $h(x,y)$ is also real and symmetric.

[**5**] (Computational Project) **Fourier Spectrum and Average Value**

(a) Use in Matlab "help fft" and "help fft2" to learn the commands for computing discrete Fourier transforms. Sample codes using the Fourier transform in 1D and 2D are posted on the class webpage.

(b) Download Fig5.26a and compute its (centered) Fourier spectrum.

(c) Display the spectrum.

(d) Using your algorithm, obtain the average value of the input image.

Remarks on notations

• We will use x, y for the spatial variables, and u, v for the frequency variables, in both continuous and discrete settings.

• The standard notation for the transform $F(u)$ of a function $f(x)$ is $\hat{f}(u)$ (we will not use it here; we will use $F(u)$).

• The textbook uses $F^*(u, v)$ for the complex conjugate:

$$F^*(u, v) = \overline{F(u, v)}$$

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