Math 155, Vese

• Midterm on Monday, February 9, 2009, 1.00-1.50pm (lecture room)

• Sections covered for the midterm: 1.4, 2.3.4, 2.4 (2.4.1-2.4.4), 3.1, 3.2 (except bit-plane slicing), 3.3, 3.4, 3.5, 3.6, 4.2.1, 4.2.2

• Solutions to exercises marked with an asterisk are posted on the authors web-page, at http://www.imageprocessingplace.com/root_files_V3/problem_solutions.htm

• Additional office hours with the instructor on Friday, February 6, at MS 7620-D: 2-3pm and 4-5pm.

Homework # 4   Due on Friday, February 6

[**1**] The median, ξ, of a set of numbers is such that half the values in the set are less than or equal to ξ, and half are greater than or equal to ξ. For example, the median of the set of values $\{2, 3, 8, 20, 21, 25, 31\}$ is 20. Show that an operator applied to the set of images (matrices) of the same dimension, that computes the median, is nonlinear.

[**2**] Write a computer program that will denoise an image using the 3x3 median filter. Apply your algorithm to the X-Ray image of circuit board corrupted by salt-and-pepper noise (Fig3.37(a).jpg). You should turn in the details of the method, your computer program, the input and output images. Perform your calculations only for interior pixels, not for boundary pixels. Explain your result and compare it with the output obtained in the previous homework on the same image (using a linear average filter).

[**3**] Write a computer program that implements the operation $g(x, y) = f(x, y) - \nabla^2 f(x, y)$, in the form of a spatial linear filter with a 3x3 mask. Give the form of the mask and apply the program to the image of the North Pole of the moon (Fig3.40(a).jpg). You should turn in the details of the method, the computer program, the input and output images. Perform your calculations only for interior pixels, not for boundary pixels. Explain your result.

[**4**] Show that the continuous Laplacian is a linear operation, in other words show that the mapping $f \mapsto \nabla^2 f$ is linear on the vector space of functions $f \in C^2$ in two dimensions.

[**5**] (a) Show using Taylor's expansion that the finite differences formula $\frac{f(x+h)-f(x-h)}{2h}$ is a second order approximation of the first-order derivative $f'(x)$.

(b) Apply this formula to approximate the gradient map

$$g(x, y) = |\nabla f|^2(x, y) = \Big(\frac{\partial f(x, y)}{\partial x}\Big)^2 + \Big(\frac{\partial f(x, y)}{\partial y}\Big)^2.$$

(c) Download Fig5.26a and plot an image negative of its gradient map g (edges will appear black, while homogenous regions will appear white; rescaling may be necessary). Explain the steps taken. Ignore the pixels on the boundary of the image for simplicity, when computing the discrete gradient.