Math 155, Vese
Homework # 3 Due Friday January 30

[1] Show that the Laplacian operation $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ is isotropic (invariant under rotations, or rotationally invariant). You will need the following equations relating coordinates after axis rotation by an angle θ:

$$x = x' \cos\theta - y' \sin\theta$$

$$y = x' \sin\theta + y' \cos\theta$$

where (x, y) are the unrotated and (x', y') are the rotated coordinates.

[2]

(a) Show that the magnitude of the gradient $|\nabla f| = \sqrt{(f_x)^2 + (f_y)^2}$ is an isotropic operation.

(b) Show that the isotropic property is lost in general if the gradient magnitude is approximated by $|\nabla f| \approx |f_x| + |f_y|$.

[3] *Computational project: Spatial filtering* Consider the noisy X-ray image of circuit board corrupted by salt-and-pepper noise. Filter this image by applying a linear average filter with a 3×3 mask (use the average mask with entries $w_{s,t} = \frac{1}{9}$, for all $s, t \in \{-1, 0, 1\}$). You can keep the border pixels unchanged.