Math 155

Homework # 2   Due on Friday, January 23

[1] Give a single intensity transformation function T for spreading the intensities of an image so the lowest intensity is 0 and the highest is $L-1$.

[2] (Histogram equalization in continuous variables) An image has the gray-level PDF

$$p_r(r) = \begin{cases} \frac{6r+2}{3(L-1)^2+2(L-1)} & \text{if } 0 \leq r \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

with $L-1>0$.

(a) Verify some of the properties that a PDF has to satisfy: $p_r(r) \geq 0$ for all $r \in (-\infty, \infty)$ and $\int_{-\infty}^{\infty} p_r(r)dr = 1$.

(b) Find the tranformation function $s = T(r)$ obtained through "histogram equalization" in continuous variables.

(c) Verify that $p_s(s)$ is a uniform "flat" distribution for $s \in [0,1]$ (recall the formula $p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$).

[3] (Histogram matching in continuous variables) An image has the gray-level PDF $p_r(r) = -2r+2$, with $0 \leq r \leq 1$. It is desired to transform the gray levels of this image so that they will have the specified $p_z(z) = 2z$, $0 \leq z \leq 1$. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this (here $L-1=1$).

[4] A linear spatial filter of size $(2a+1) \times (2b+1)$ defined by the transformation H, $g = H[f]$, is given by

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t),$$

where $f(x,y)$ is a given input image, a, b are positive integers, and $w(s,t)$ are weights for $-a \leq s \leq a$, $-b \leq t \leq b$.

(a) Give the definition of a linear transformation $H : V \to V$, where V is a vector space.

(b) Show that H defined above is indeed a linear transformation (assume images defined on the entire plane, or ignore border effects).