Math 155, Vese: Homework # 8, due on Friday, March 9

[1] Start with equation (5.4-19) and derive equation (5.4-21).

[2] Consider the motion blur in the frequency domain given by

$$H(u,v) = \int_0^T e^{-2\pi i[ux_0(t)+vy_0(t)]}dt.$$

For uniform motion given by $x_0(t) = \frac{at}{T}$ and $y_0(t) = \frac{bt}{T}$ (T=exposure time), show that the degradation function becomes

$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin\left[\pi(ua+vb)\right]e^{-\pi i(ua+vb)}.$$

[3] Consider a linear, position-invariant image degradation system with impulse response

$$h(x-\alpha, y-\beta) = e^{-[(x-\alpha)^2+(y-\beta)^2]}.$$

Suppose that the input to the system is an image consiting of a line of infinitesimal width located at $x = a$, and modeled by $f(x,y) = \delta(x-a)$, where δ is the impulse function. Assuming no noise, what is the output image $g(x,y)$?

Hint: Use equation (5.5-13) and also that

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}\, dz = 1,$$

where μ and σ are constants.

[4]

(a) Consider the radial function $h_0(r) = e^{-r^2/2\sigma^2}$, where $r^2 = x^2+y^2$. Compute the 2D Laplacian $\nabla^2[h_0(\sqrt{x^2+y^2})]$.

(b) A certain X-ray imaging geometry produces a blurring degradation that can be modeled as the convolution of the sensed image with spatial, circularly symmetric function

$$h(r) = \Big[(r^2-2\sigma^2)/\sigma^4\Big]e^{-r^2/2\sigma^2},$$

where $r^2 = x^2+y^2$. Show that the degradation in the frequency domain is given by the expression

$$H(u,v) = -2\pi\sigma^2(u^2+v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}.$$

[5] Parametric Wiener Filter

(a) Implement a motion blurring filter as in problem [2] above.

(b) Blur image 5.26(a) in the +45o direction using $T = 1$, as in Fig. 5.26(b) ($a = b = 0.1$).

(c) Add a small amount of Gaussian noise of 0 mean to the blurred image. For this, you can use the Matlab command:

J = imnoise(I,'gaussian',0,v) adds Gaussian white noise of mean 0 and variance v to the image I (the parameter v needs to be specified). The default is zero mean noise with 0.01 variance.

(d) Restore the image using the parametric Wiener filter given by

$$\hat{F}(u,v) = \Big[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\Big]G(u,v),$$

where K is a specified constant, chosen to obtain best visual results.

(note that you may have to avoid division by zero in the above formula).