Math 155, Vese

Homework # 4 Due on Friday, February 9

[1] Recall that the 1D DFT is

$$F(u) = \frac{1}{M} \sum_{x=0}^{M-1} f(x) e^{-2\pi iux/M}.$$

Show the identity

$$f(x) = \sum_{u=0}^{M-1} F(u) e^{2\pi iux/M},$$

using the following orthogonality of exponentials

$$\sum_{u=0}^{M-1} e^{-2\pi iuy/M} e^{2\pi iux/M} = \begin{cases} M & \text{if } x = y \\ 0 & \text{otherwise.} \end{cases}$$

[2] Show that the continuous 2D Fourier transform is a linear process.

[3] Compute in continuous variables the Fourier transform of the function

$$f(x) = \begin{cases} A, \text{ if } 0 \leq x \leq K, \\ 0, \text{ otherwise,} \end{cases}$$

where A and K are positive constants. Evaluate $F(0)$.

[4] Consider again the 2D continous Fourier transform and its inverse (denote by $H(u, v)$ the 2D Fourier transform of the spatial filter $h(x, y)$). Show that if the transform $H(u, v)$ is real and symmetric, i.e. if

$$H(u, v) = \overline{H(u, v)} = \overline{H(-u, -v)} = H(-u, -v),$$

then the corresponding spatial domain filter $h(x, y)$ is also real and symmetric.

[5] Fourier Spectrum and Average Value
(a) Download Fig5.26a and compute its (centered) Fourier spectrum.
(b) Display the spectrum.
(c) Use your result in (a) to compute the average value of the image.