Math 155, Vese

Note: Exceptionally there is no office hour with the instructor on Monday, January 29.

Homework # 3   Due on Friday, February 2

[**1**] Write a computer program that will denoise an image using the 3x3 median filter. Apply your algorithm to the X-Ray image of circuit board corrupted by salt-and-pepper noise (Fig3.37(a).jpg). You should turn in the details of the method, your computer program, the input and output images. Perform your calculations only for interior pixels, not for boundary pixels. Compare your result with the output obtained by applying a linear average filter with a 3×3 mask to the same noisy image (use the average mask $w_{s,t} = \frac{1}{9}$ for all $s, t \in \{-1, 0, 1\}$). Explain your results.

[**2**] Write a computer program that implements the operation $g(x, y) = f(x, y) - \nabla^2 f(x, y)$, in the form of a spatial linear filter with a 3x3 mask. Give the form of the mask and apply the program to the image of the North Pole of the moon (Fig3.40(a).jpg). You should turn in the details of the method, the computer program, the input and output images. Perform your calculations only for interior pixels, not for boundary pixels. Explain your result.

[**3**] In a given application, an averaging mask is applied to input images to reduce noise, and then a Laplacian mask is applied to enhance small details. Would the result be the same if the order of these operations were reversed ? Explain.

[**4**]

(a) Show using Taylor's expansion that the finite differences expression $\frac{f(x+h)-f(x-h)}{2h}$ is a second order approximation of the first-order derivative $f'(x)$.

(b) Apply this formula to approximate the gradient map

$$g(x, y) = |\nabla f|^2(x, y) = \Big(\frac{\partial f(x, y)}{\partial x}\Big)^2 + \Big(\frac{\partial f(x, y)}{\partial y}\Big)^2.$$

(c) Download Fig5.26a and plot an image negative of its gradient map g (edges will appear black, while homogenous regions will appear white). Explain the steps taken. Ignore the pixels on the boundary of the image for simplicity, when taking the discrete gradient.