Math 155

Homework # 2   Due on Friday, January 26

[**1**] An image has the gray level PDF $p_r(r) = -2r + 2$, with $0 \leq r \leq 1$. It is desired to transform the gray levels of this image so that they will have the specified $p_z(z) = 2z$, $0 \leq z \leq 1$. Assume continuous quantities and find the transformation (in terms of r and z) that will accomplish this.

[**2**] The median, ξ, of a set of numbers is such that half the values in the set are below ξ and the other half are above it. For example, the median of the set of values $\{2, 3, 8, 20, 21, 25, 31\}$ is 20.
   (a) Recall the definition of a linear operator H.
   (b) Show that an operator that computes the median is nonlinear.

[**3**] Show that the Laplacian operation $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ is isotropic (invariant under rotations, or rotationally invariant). You will need the following equations relating coordinates after axis rotation by an angle θ:

$$x = x' \cos \theta - y' \sin \theta$$

$$y = x' \sin \theta + y' \cos \theta$$

where (x, y) are the unrotated and (x', y') are the rotated coordinates.

[**4**]
   (a) Show that the magnitude of the gradient $|\nabla f| = \sqrt{(f_x)^2 + (f_y)^2}$ is an isotropic operation.
   (b) Show that the isotropic property is lost in general if the gradient magnitude is approximated by $|\nabla f| \approx |f_x| + |f_y|$.

Optional There is no computational assignment for this homework. Those interested, can try to formulate and solve a computational project using the histogram specification, or using the enhancement method from pages 105-106. If needed, I can provide the input images for these experiments.