Math 155, Vese

• **REMINDER:** One hour final written exam on Friday, June 9, time 1-2pm. All sections are covered for the final, but more questions will be given from the second part of the course.

Sections covered for the midterm: 2.3.4, 2.4 (except 2.4.4), 3.1, 3.2, 3.3, 3.4.1, 3.4.2, 3.5, 3.6, 3.7, Chapter 4 (except 4.3.4, 4.6.3, 4.6.6, 4.6.7).

Sections covered after the midterm: 4.6.4, 5.1-5.11 (except "Adaptive Median Filter"), 6.2.1, 6.4, 6.5.1, 6.5.5, 6.6.1, 6.6.2, 6.7.3, and 10.1, 10.2.1, 10.2.2.

Review session with the instructor: Thursday, 3-5pm, MS 6221.

Homework # 9, due on Friday, June 9

(if needed, you can turn in the assignment Friday afternoon, after 2pm, directly to the TA or to the instructor; assignments turned in on Monday, June 12 may not be graded).

[**1**] Consider a linear, position-invariant image degradation system with impulse response

$$h(x-\alpha, y-\beta) = e^{-[(x-\alpha)^2+(y-\beta)^2]}.$$

Suppose that the input to the system is an image consiting of a line of infinitesimal width located at $x = a$, and modeled by $f(x,y) = \delta(x-a)$, where δ is the impulse function. Assuming no noise, what is the output image $g(x,y)$?

Hint: Use equation (5.5-13) and also that

$$\int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}\, dz = 1,$$

where μ and σ are constants.

[**2**]

(a) Consider the radial function $h_0(r) = e^{-r^2/2\sigma^2}$, where $r^2 = x^2+y^2$. Compute the 2D Laplacian $\nabla^2[h_0(\sqrt{x^2+y^2})]$.

(b) A certain X-ray imaging geometry produces a blurring degradation that can be modeled as the convolution of the sensed image with spatial, circularly symmetric function

$$h(r) = \Big[(r^2-2\sigma^2)/\sigma^4\Big] e^{-r^2/2\sigma^2},$$

where $r^2 = x^2 + y^2$. Show that the degradation in the frequency domain is given by the expression

$$H(u,v) = -2\pi\sigma^2(u^2+v^2)e^{-2\pi^2\sigma^2(u^2+v^2)}.$$

[3] Suppose that, instead of using quadrilaterals, you use triangular regions in Section 5.11 to establish a geometric spatial transformation and gray-level interpolation. What would be the equations analogous to equations (5.11-5), (5.11-6), and (5.11-7) for triangular regions ?

[4]
(a) Give the main steps of edge finding using the zero-crossings of the Laplacian of Gaussian (as in Example 10.5).
(b) Implement and apply this method to the angiogram image from Figure 10.15(a), as in example 10.5 (do not compare with the Sobel gradient).

[5] Color Edge Detection (THIS PROBLEM IS OPTIONAL)
Download the color image of Lena Fig6.38a.jpg. Compute at each interior point (x, y) the "value of the rate of change" given by $F(\theta)$ (see Section 6.7.3). Thus obtain an edge map of the image by plotting $F(\theta)$, as in Figure 6.46 (b).
- At every interior point (x, y), use the following central approximations to the 1st order partial derivatives:

$$\frac{\partial f(x,y)}{\partial x} \approx (f(x+1,y) - f(x-1,y))/2,$$

$$\frac{\partial f(x,y)}{\partial y} \approx (f(x,y+1) - f(x,y-1))/2.$$

- Use as usual the "imread" Matlab command
f=imread('Fig6.38a.jpg');
[M N P]=size(f);
This will produce a vector-valued function f of dimensions $M = 512$, $N = 512$, and $P = 3$ (the red channel is given by $f(i,j,1)$, the green channel by $f(i,j,2)$ and the blue channel by $f(i,j,3)$, with $i = 1,...,M$ and $j = 1,...,N$ being the spatial dimensions of the color image).
- Do not include in your calculations the boundary points, for simplicity.