Math 155, Vese

Homework # 5 Due on Friday, May 28

[1] Show that the Fourier transform of the 2-D continuous sine function $f(x, y) = A\sin(u_0x + v_0y)$ is the pair of conjugate impulses

$$F(u,v) = -i\frac{A}{2}\Big[\delta\Big(u - \frac{u_0}{2\pi}, v - \frac{v_0}{2\pi}\Big) - \delta\Big(u + \frac{u_0}{2\pi}, v + \frac{v_0}{2\pi}\Big)\Big].$$

Hint: use the continuous version of the FT, and express the sine in terms of exponentials.

[2] Consider the motion blur

$$H(u,v) = \int_0^T e^{-2\pi i[ux_0(t)+vy_0(t)]}dt.$$

For uniform motion given by $x_0(t) = \frac{at}{T}$ and $y_0(t) = \frac{bt}{T}$ (T=exposure time), show that the degradation function becomes

$$H(u,v) = \frac{T}{\pi(ua + vb)}\sin[\pi(ua + vb)]e^{-\pi i(ua+vb)}.$$

[3] Periodic Noise Reduction Using a Notch Filter

(a) Write a program that implements sinusoidal noise of the form given in Problem [1] above. The inputs to the program must be the amplitude, A, and the two frequency components u0 and v0 shown in the problem equation.

(b) Download image 5.26(a) and add sinusoidal noise to it, with u0 = M/2 (the image is square) and v0 = 0. The value of A must be high enough for the noise to be quite visible in the image.

(c) Compute and display the spectrum of the image.

(d) Notch-filter the image using a notch filter of the form shown in Fig. 5.19(c).

[4] Parametric Wiener Filter

(a) Implement a motion blurring filter as in problem [2] above.

(b) Blur image 5.26(a) in the +45o direction using $T = 1$, as in Fig. 5.26(b) ($a = b = 0.1$).

(c) Add Gaussian noise of 0 mean to the blurred image.

(in Matlab use the following commands to add Gaussian noise of zero mean to an image:

```
f=imread('Fig5.26a.jpg');
imagesc(f); colormap(gray);
f=double(f);
[M N]=size(f);
noise=randn(256,256);
g=f+10*noise;
imagesc(g); colormap(gray);
)
```

(d) Restore the image using the parametric Wiener filter given by

$$\hat{F}(u,v) = \Big[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\Big]G(u,v).$$

(in the D.S. on Tuesday you may discuss how to avoid division by zero, if $H(u,v)=0$).