Math 155, Spring 2004, Vese

Homework # 2 Due on Friday, April 30

[1]

(a) Give a continuous function for implementing the contrast stretching transformation shown in Fig. 3.2(a). In addition to m, your function must include a parameter, E, for controlling the slope of the function as it transitions from low to high gray-level values. Your function should be normalized so that its minimum and maximum values are 0 and 1, respectively.

(b) Sketch (plot) a family of transformations function of E, for a fixed value $m = L/2$, where L is the number of gray levels in the image. Include the transformation that will output a binary image.

[2] Suppose that a digital image is subjected to histogram equalization. Show that a second pass of histogram equalization will produce exactly the same result as the first pass.

[3] Show that the Laplacian operation $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$ is isotropic (invariant under rotations, or rotationally invariant). You will need the following equations relating coordinates after axis rotation by an angle θ:

$$x = x' \cos\theta - y' \sin\theta$$

$$y = x' \sin\theta + y' \cos\theta$$

where (x, y) are the unrotated and (x', y') are the rotated coordinates.

[4]

(a) Show that the magnitude of the gradient $|\nabla f| = \sqrt{(f_x)^2 + (f_y)^2}$ is an isotropic operation.

(b) Show that the isotropic property is lost in general if the gradient magnitude is approximated by $|\nabla f| \approx |f_x| + |f_y|$.

[5] Write a computer program that will denoise an image using the 3x3 median filter. Apply your algorithm to the X-Ray image of circuit board corrupted by salt-and-pepper noise (Fig3.37(a).jpg). You should turn in the details of the method, your computer program, the input and output images. Perform your calculations only for interior pixels, not for boundary pixels. Explain your result.

[6] Write a computer program that implements the operation $g(x, y) = f(x, y) - \nabla^2 f(x, y)$, in the form of a spatial linear filter with a 3x3 mask. Give the form of the mask and apply the program to the image of the North Pole of the moon (Fig3.40(a).jpg). You should turn in the details of the method, the computer program, the input and output images. Perform your calculations only for interior pixels, not for boundary pixels. Explain your result.