

## Math 155

Final exam: Tuesday, March 21st, 3-6pm, MS 5118 (usual lecture room).

Additional office hours:

Monday, March 20:

- 10am-12pm with Baichuan
- 1-2pm with Vese.

**Sample questions** (but not limited to this list):

- What is the definition of
  - the arithmetic mean filter ?
  - the geometric mean filter ?
  - the harmonic mean filter ?
  - the contraharmonic mean filter ?
  - the median filter ?
  - the max and mean filters ?
  - the midpoint filter ?
  - the alpha-trimmed mean filter ?
- Give the main steps of an adaptive, local noise reduction filter ? (explain).
- Given examples of low-pass and high-pass filters and explain.
- Give the main steps of filtering in the frequency domain.
- Prove properties of the FT like those done in class or in the homework.
- What is the goal of a bandreject filter for periodic noise reduction ? Give an example. What is the goal of a bandpass filter ? Give an example.
- Explain the optimum notch filtering technique.
- Give the definition of a unite impulse, located at coordinates  $(x, y)$ .
- Show that, if  $g(x, y) = H[f(x, y)]$ , with  $H$  linear, position invariant, and extending the additivity property to integrals, then  $H$  is given by a convolution with the function  $f$ .
- Show in continuous variables that the Fourier transform of

$$f(x, y)e^{2\pi i(u_0x+v_0y)}$$

is given by  $F(u - u_0, v - v_0)$ , where  $F(u, v)$  is the Fourier transform of  $f(x, y)$ .

- Consider the motion degradation function

$$g(x, y) = \int_0^T f[x - x_0(t), y - y_0(t)]dt.$$

Find  $H(u, v)$  such that  $G(u, v) = H(u, v)F(u, v)$ , where  $G$  and  $F$  are the Fourier transforms of  $g, f$ , respectively (show the details of the derivation).

- Give an example of degradation function  $h$  and explain how it is applied to the original image  $f$  ?

- What is the linear degradation model in the spatial domain and frequency domain ?

- Give an example of a restoration (deconvolution) model.

- Give an example of a mask  $w$  that can be used to detect:

- a light point on a constant dark background (explain)

- a light vertical line on a constant dark background (explain), etc.

- How can you detect edges using the gradient  $\nabla^2 f$  of an image  $f(x, y)$

? Give an example of approximation to the gradient.

- Give the main steps of the zero-crossings method for edge detection (Marr-Hildreth edge detector).

- Recall the definition in continuous variables of the convolution

$$f(x, y) * g(x, y).$$

- Show the convolution Thm.

- Show that  $\nabla^2(f * g) = f * (\nabla^2 g)$  at  $(x, y)$ , where  $\nabla^2$  denotes the Laplace operator in  $(x, y)$ .

See additional problems and exercises from the homework.