

Math 151A HW #2; due on Friday, April 20

PLEASE WRITE THE SECTION NUMBER ON YOUR HOMEWORK

#1(ab) Use algebraic manipulations to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where

$$f(x) = x^4 + 2x^2 - x - 3.$$

(a) $g_1(x) = (3 + x - 2x^2)^{1/4}$

(b) $g_2(x) = \left(\frac{x+3-x^4}{2}\right)^{1/2}$

#2

(a) Perform four iterations, if possible, on each of the functions g defined in Exercise 1 (only (a) and (b)). Let $p_0 = 1$ and $p_{n+1} = g(p_n)$, for $n = 0, 1, 2, 3$.

(b) Which function do you think gives the best approximation to the solution ?

#7 Use the Fixed Point Theorems from Section 2.2 to show that $g(x) = \pi + 0.5 \sin(x/2)$ has a unique fixed point on $[0, 2\pi]$. Use fixed-point iteration to find an approximation to the fixed point that is accurate to within 10^{-2} . Use the theoretical result to estimate the number of iterations required to achieve 10^{-2} accuracy, and compare this theoretical estimate to the number actually needed.

#9 Use a fixed-point iteration method to find an approximation to $\sqrt{3}$ that is accurate to within 10^{-4} . Compare your result and the number of iterations required with the answer obtained using the Bisection Algorithm from the previous homework.

#19(a) Use the Fixed Point Theorems from Section 2.2 to show that the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{1}{x_{n-1}}, \quad \text{for } n \geq 1,$$

converges to $\sqrt{2}$ whenever $x_0 > \sqrt{2}$.