UCLA MATH 151A/3, Monday October 29, 2001
MIDTERM EXAM

NAME ______________________________ **STUDENT ID #** ____________________

This is a closed-book and closed-note examination. Please show all your work. Partial credit will be given to partial answers. There are 4 problems of total 100 points.

PROBLEM	1	2	3	4	TOTAL
SCORE					

1.

(a) Do three iterations (by hand) of the bisection method, applied to $f(x) = x^3 - 2$, using $a = 0$ and $b = 2$ (find p_0, p_1 and p_2).

(b) How many iterations does the theory predict it will take to achieve 10^{-5} accuracy, to approximate the root of $x^3 - 2 = 0$ by the bisection method on the interval $[0, 2]$?

2.

(a) By a theorem from the course, show that the function $g(x) = 1 + e^{-x}$ has a unique fixed point on $[1, 2]$ (given values: $e^{-1} = 0.3679$, $e^{-2} = 0.1353$).

(b) For $p_0 = 1$, compute p_1 using the fixed-point iteration.

(c) How many iterations does the theory predict it will take to achieve 10^{-5} accuracy, to approximate the fixed point, starting with $p_0 = 1$?

3.

(a) Write down Newton's iteration as applied to the function $f(x) = x^3 - 2$. Simplify the formula as much as possible.

(b) Write down the Secant iteration as applied to the same function $f(x) = x^3 - 2$.

4.

(a) Construct the Lagrange polynomial interpolating the points $x_0 = 0$ and $x_1 = h$, to approximate the function $f(x) = \cos x$, where $h > 0$ is a small parameter.

(b) Write down the error formula as applied to the above interpolation. Find an upper bound for the error on the interval $[0, h]$, function of the parameter h.

(c) Construct the Lagrange polynomial interpolating the points $x_0 = 0$, $x_1 = h$, and $x_2 = 2h$ to approximate the function $f(x) = \cos x$.