

UCLA MATH 151A/2, Friday February 9, 2001

NAME \_\_\_\_\_ STUDENT ID # \_\_\_\_\_

This is a closed-book and closed-note examination. No calculators are allowed. Please show all your work. Partial credit will be given to partial answers. There are 4 problems of total 20 points.

PROBLEM	1	2	3	4	TOTAL
SCORE					

I. Let  $f(x) = \sqrt{\pi x} - \cos(\pi x)$ .

- (a) Prove that the equation  $f(x) = 0$  has at least a solution  $p$  in the interval  $[0, 1]$ .
- (b) By the Bisection method, find  $p_n$ ,  $n \leq 2$  on  $[0, 1]$ . Use the table

$x$	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1
$f(x)$	-1	-0.297	0.179	0.702	1.253	1.783	2.242	2.581	2.772

and write your answers in the next table.

$n$	$a_n$	$b_n$	$p_n$	$f(p_n)$
0	0	1		
1				
2				

- (c) How many iterations are necessary to solve  $\sqrt{\pi x} - \cos(\pi x) = 0$  with accuracy  $10^{-5}$  on  $[0, 1]$ ?

II.

- (a) Use the Theorem from the course to prove that  $g(x) = \pi + 0.5 \sin \frac{x}{2}$  has a unique fixed point on  $[0, 2\pi]$ .
- (b) For  $p_0 = \pi$ , compute  $p_1$ .
- (c) How many iterations are necessary to achieve the accuracy  $10^{-2}$  ?

III. Let  $f(x) = x^2 - 6$ .

(a) Prove that  $f$  has a positive zero  $p$ .

(b) With  $p_0 = 1$ , find  $p_2$  by Newton's iterations.

IV. Let  $f(x) = \cos x$ ,  $x_0 = 0$ ,  $x_1 = 0.6$  and  $x_2 = 0.9$ .

(a) Construct an interpolation polynomial of degree at most two to approximate  $f$ . Use  $\cos(0.6) = 0.8$  and  $\cos(0.9) = 0.6$ .

(b) Use the Theorem of the course to find an error bound for the approximation.