Math 151A
HW #2, due Friday, January 19

- Reading: sections 2.1, 2.2.
- The problems below from Section 2.2.

#1(ab) Use algebraic manipulation to show that each of the following functions has a fixed point at \( p \) precisely when \( f(p) = 0 \), where \( f(p) = x^4 + 2x^2 - x - 3 \).

(a) \( g_1(x) = (3 + x - 2x^2)^{1/4} \)

(b) \( g_2(x) = \left( \frac{x + 3 - x^4}{2} \right)^{1/2} \)

#2
(a) Perform four iterations, if possible, on each of the functions \( g \) defined in Exercise 1 (only (a) and (b)). Let \( p_0 = 1 \) and \( p_{n+1} = g(p_n) \), for \( n = 0, 1, 2, 3 \).

(b) Which function do you think gives the best approximation to the solution?

#7 Use Thm. 2.2 to show that \( g(x) = \pi + 0.5 \sin(x/2) \) has a unique fixed point on \([0, 2\pi]\). Use fixed-point iteration to find an approximation to the fixed point that is accurate to within \( 10^{-2} \). Use Corollary 2.4 to estimate the number of iterations required to achieve \( 10^{-2} \) accuracy, and compare this theoretical estimate to the number actually needed.

#9 Use a fixed-point iteration method to find an approximation to \( \sqrt{3} \) that is accurate to within \( 10^{-4} \). Compare your result and the number of iterations required with the answer obtained using the Bisection Algorithm from the previous homework.

#19(a) Use Thm. 2.3 to show that the sequence defined by

\[ x_n = \frac{1}{2} x_{n-1} + \frac{1}{x_{n-1}}, \quad \text{for } n \geq 1, \]

converges to \( \sqrt{2} \) whenever \( x_0 > \sqrt{2} \).

Note: The code is not provided this time. You may want to write your own fixed-point iteration code in the language of your choice for the longer calculations. Please include your code with the solutions.