Math 151A
HW #2, due Friday, January 19

- Reading: sections 2.1, 2.2.
- The problems below from Section 2.2.

#1(ab) Use algebraic manipulation to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where $f(p) = x^4 + 2x^2 - x - 3$.
(a) $g_1(x) = (3 + x - 2x^2)^{1/4}$
(b) $g_2(x) = \left(\frac{x+3-x^4}{2}\right)^{1/2}$

#2
(a) Perform four iterations, if possible, on each of the functions g defined in Exercise 1 (only (a) and (b)). Let $p_0 = 1$ and $p_{n+1} = g(p_n)$, for $n = 0, 1, 2, 3$.
(b) Which function do you think gives the best approximation to the solution ?

#7 Use Thm. 2.2 to show that $g(x) = \pi + 0.5\sin(x/2)$ has a unique fixed point on $[0, 2\pi]$. Use fixed-point iteration to find an approximation to the fixed point that is accurate to within 10^{-2}. Use Corollary 2.4 to estimate the number of iterations required to achieve 10^{-2} accuracy, and compare this theoretical estimate to the number actually needed.

#9 Use a fixed-point iteration method to find an approximation to $\sqrt{3}$ that is accurate to within 10^{-4}. Compare your result and the number of iterations required with the answer obtained using the Bisection Algorithm from the previous homework.

#19(a) Use Thm. 2.3 to show that the sequence defined by

$$x_n = \frac{1}{2}x_{n-1} + \frac{1}{x_{n-1}}, \qquad \text{for } n \geq 1,$$

converges to $\sqrt{2}$ whenever $x_0 > \sqrt{2}$.

Note: The code is not provided this time. You may want to write your own fixed-point iteration code in the language of your choice for the longer calculations. Please include your code with the solutions.