Final Exam, Math 151A/3, Fall 2001, UCLA, 12/11/2001, 8am-11am

NAME:

STUDENT ID #:

This is a closed-book and closed-note examination. Please show all you work. Partial credit will be given to partial answers.

There are 8 problems of total 100 points.

Time: 3 hours.

SCORE:

I ________

II ________

III ________

IV ________

V ________

VI ________

VII ________

VIII ________

Total ____________

I Construct in 3 different ways the Lagrange interpolating polynomial for the following data:

x	$f(x) = xe^x$
$x_0 = 0$	0
$x_1 = 1$	e
$x_2 = 2$	$2e^2$

Method 1: by the definition of the Lagrange polynomial.
Method 2: by Neville's method.
Method 3: by Newton's interpolatory divided-difference formula.

II For the data in problem I, write the error formula, and find an upper bound for the absolute error, for $x = 0.5$.

III

1. Giving two points x_0 and $x_0 + h$, with $h > 0$, and a function $f \in C^2[x_0, x_0 + h]$, derive an approximation to $f'(x_0)$, with error term.
2. We consider again the data from problem I:

x_i	$f(x_i)$
$x_0 = 0$	0
$x_1 = 1$	e
$x_2 = 2$	$2e^2$

Give:

One approximation to $f'(0)$
Three different approximations to $f'(1)$
One approximation to $f''(1)$
One approximation to $f'(2)$

IV Find the constants c_0, c_1 and x_1 so that the quadrature formula

$$\int_0^1 f(x)dx = c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision. What is that degree of precision ?

V The Trapezoidal rule applied to $\int_0^2 f(x)dx$ gives the value 4, and Simpson's rule gives the value 2. What is $f(1)$?

VI Consider $f \in C^2[a, b]$, $h = \frac{b-a}{n}$, $x_j = a + jh$, $j = 0, 1, ..., n$, and $\mu \in (a, b)$. The error term in the Composite Trapezoidal rule is:

$$-\frac{b-a}{12}h^2 f''(\mu).$$

(a) Determine the values of n and h required to approximate $\int_0^2 xe^x dx$ to within 10^{-4}, by the Composite Trapezoidal rule.

(b) Write the approximation formula for $n = 4$.

VII Solve the following linear system $Ax = b$, if possible, by Gaussian elimination with backward substitution, and determine whether row interchanges are necessary.

$$\begin{array}{rcrcrcrcr}
x_1 & - & x_2 & + & 2x_3 & - & x_4 & = & -8 \\
2x_1 & - & 2x_2 & + & 3x_3 & - & 3x_4 & = & -20 \\
x_1 & + & x_2 & + & x_3 & & & = & -2 \\
x_1 & - & x_2 & + & 4x_3 & + & 3x_4 & = & 4.
\end{array}$$

VIII For A the matrix coefficient of the linear system from problem VII, find a 4x4 permutation matrix P, such that PA can be factorized into LU, with L a lower triangular matrix with entries 1 on the diagonal, and U an upper triangular matrix. Find the matrices L and U.

Without showing the details, what are the steps for solving the same system $Ax = b$, using the factorization $PA = LU$?