Math 151A Final Exam

March 19, 11:30 am -2:30 pm, 2001 @UCLA

PRINT NAME: ____

I acknowledge and accept the honor code.

SIGN NAME: ____

- No books
- No notes
- No calculators
- 10 problems
- 100 points
- Please do any 8 problems
- 180 minutes
- *Show work*
- Good Luck!

SCORE:

1	________
2	________
3	________
4	________
5	________
6	________
7	________
8	________
9	________
10	________
Total	________

1. (10 points) Short answer questions:

   a Give the relation between errors e_{n+1} and e_n, where $e_n := |p_n - p|$, associated with Newton's method for solving non-linear equations. (You can use a constant C to represent any derivative which occur in this relation.)

   b As an approximation method, is numerical differentiation stable? why or why not?

   c A linear system $Ax = 0$ has a unique solution $x = 0$, does the system $Ax = b$ has unique solution for any given b?

   d Assume a given function $f \in C[a, b]$. How many points would you use to construct a Lagrange polynomial of degree 10?

2. (10 points) Newton's method is used to find a solution to $p = p - f(p)/f'(p)$ based on an initial approximation p_0. Given the tolerance TOL and the maximum number of iterations N_0, please write the algorithm for this method.

3. (10 points) For $f = x^4$, let $x_0 = 0$, $x_1 = 1$, and $x_2 = 2$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(0.5)$, and find the actual error.

4. (10 points) (a)Let $(i, j) \in \{0, 1, \cdots n\}^2$ be two distinct integers. Express $P_{0,1\cdots,n} := P_{x_0,x_1,\cdots,x_n}$ in terms of $P_{0,\cdots i-1,i+1\cdots n}$ and $P_{0,\cdots j-1,j+1\cdots n}$.

   (b) Suppose $x_j = j$ for $j = 0, 1, 2, 3$ and it is known that

$$P_{0,1}(x) = x + 1, P_{1,2}(x) = 3x - 1, \quad \text{and} \quad P_{1,2,3}(1.5) = 4.$$

   Find $P_{0,1,2,3}(1.5)$.

5. (10 points) (a) Express the interpolating polynomial $P(x)$ using Newton's forward divided difference formula.

   (b) For a function f the forward divided-difference are given by

$$
\begin{array}{llll}
x_0 = 0 & f(x_0) & & \\
 & & f[x_0, x_1] & \\
x_1 = 0.4 & f(x_1) & & f[x_0, x_1, x_2] = 50/7 \\
 & & f[x_1, x_2] = 10 & \\
x_2 = 0.7 & f(x_2) = 6 & &
\end{array}
$$

Determine the missing entries in the table.

6. (10 points)Analyze the roundoff errors for

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} - \frac{h}{2}f''(\xi_0), \quad \xi_0 \in (x_0, x_0 + h).$$

Find an optimal $h > 0$ in terms of M, a bound for f'' on $(x_0, x_0 + h)$.

7. (10 points) The forward-difference formula can be expressed as

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} - \frac{h}{2}f''(x_0) - \frac{h^2}{6}f'''(x_0) + O(h^3).$$

Use extrapolation to derive an $O(h^3)$ formula for $f'(x_0)$.

8. (10 points) Determine the value of n and h required to approximate

$$\int_0^2 \frac{1}{x+4}dx$$

to within 10^{-5} and compute the approximation for $n = 4$. Use the Composite Trapezoidal rule:

$$\int_a^b f(x)dx = \frac{h}{2}[f(a) + f(b) + 2\sum_{j=1}^{n-1} f(x_j)] - \frac{b-a}{12}h^2 f''(\mu)$$

with $h = (b - a)/n$.

9. (10 points) Find the degree of accuracy of the quadrature formula

$$\int_{-1}^1 f(x)dx = f(-\frac{\sqrt{3}}{3}) + f(\frac{\sqrt{3}}{3})$$

and then derive the formula for

$$\int_a^b f(x)dx.$$

10. (10 points) (a) Obtain factorization of the form $A = LU$, where L is lower triangular with 1s on its diagonal and U is upper triangular for

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 4 & -1 \\ -2 & -3 & 4 \end{bmatrix}$$

(b) solve the linear system $Ax = b$ with $b = [2 \quad -1 \quad 1]'$.