Math 151a: HW #9. Due on Friday, June 8

[1] Approximate the integral $\int_0^1 x^2 e^{-x} dx$ using Gaussian quadrature with $n = 2$ and compare the result to the exact value obtained in Hw #7.

[2] Show that the formula $Q(P) = \sum_{i=1}^{n} c_i P(x_i)$ cannot have degree of precision greater than $2n - 1$, regardless of the choice of c_1, ..., c_n and x_1, ..., x_n. (Hint: Construct a polynomial $P(x)$ that has a double root at each of the x_i's).

[3] **(a)** Use Gaussian elimination to solve the linear system, if possible, and determine whether row interchanges were necessary:

$$\begin{aligned} 2x_1 - 1.5x_2 + 3x_3 &= 1 \\ -x_1 \qquad\quad + 2x_3 &= 3 \\ 4x_1 - 4.5x_2 + 5x_3 &= 1. \end{aligned}$$

(b) Let A be the 3x3 matrix of coefficients from the above system. From (a) obtain an LU factorization of the matrix A, if possible, and then solve the system using this factorization. (Section 6.5; Monday lecture).

[4] Consider the following matrix. Find the permutation matrix P so that PA can be factored into the product LU, where L is lower triangular with 1s on its diagonal and U is upper triangular: (Section 6.5; Monday lecture).

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

[5] Check whether the following matrix is symmetric, strictly diagonally dominant, and symmetric positive definite: (Section 6.6)

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}.$$

[6] Consider the matrix (Section 6.6)

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) Determine a lower triangular matrix L with 1s on its diagonal and D a diagonal matrix with positive diagonal entries such that $A = LDL^T$.

(b) Factor A in the form LL^T, where L is lower triangular with nonzero diagonal entries.