

**Math 151a: HW #9. Due on Friday, June 8**

[1] Approximate the integral  $\int_0^1 x^2 e^{-x} dx$  using Gaussian quadrature with  $n = 2$  and compare the result to the exact value obtained in Hw #7.

[2] Show that the formula  $Q(P) = \sum_{i=1}^n c_i P(x_i)$  cannot have degree of precision greater than  $2n - 1$ , regardless of the choice of  $c_1, \dots, c_n$  and  $x_1, \dots, x_n$ . (Hint: Construct a polynomial  $P(x)$  that has a double root at each of the  $x_i$ 's).

[3] (a) Use Gaussian elimination to solve the linear system, if possible, and determine whether row interchanges were necessary:

$$2x_1 - 1.5x_2 + 3x_3 = 1$$

$$-x_1 \quad \quad + 2x_3 = 3$$

$$4x_1 - 4.5x_2 + 5x_3 = 1.$$

(b) Let  $A$  be the 3x3 matrix of coefficients from the above system. From (a) obtain an  $LU$  factorization of the matrix  $A$ , if possible, and then solve the system using this factorization. (Section 6.5; Monday lecture).

[4] Consider the following matrix. Find the permutation matrix  $P$  so that  $PA$  can be factored into the product  $LU$ , where  $L$  is lower triangular with 1s on its diagonal and  $U$  is upper triangular: (Section 6.5; Monday lecture).

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

[5] Check whether the following matrix is symmetric, strictly diagonally dominant, and symmetric positive definite: (Section 6.6)

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}.$$

[6] Consider the matrix (Section 6.6)

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

(a) Determine a lower triangular matrix  $L$  with 1s on its diagonal and  $D$  a diagonal matrix with positive diagonal entries such that  $A = LDL^T$ .

(b) Factor  $A$  in the form  $LL^T$ , where  $L$  is lower triangular with nonzero diagonal entries.