Math 151a: HW #8. Due on Friday, June 1

[1] Use the Composite Trapezoidal, Simpson's and Midpoint Rules to approximate the integral

$$\int_1^2 x \ln(x) dx, \quad n = 4.$$

(For the Midpoint Rule use $n + 2$ subintervals.)

[2] Determine the values of n and h required to approximate

$$\int_1^2 x \ln(x) dx$$

to within 10^{-5}. Use

(a) Composite Trapezoidal Rule.
(b) Composite Simpson's Rule.
(c) Composite Midpoint Rule.

[3] Find c_1, c_2, x_1 and x_2 such that the integration formula

$$\int_{-1}^1 f(x) dx \approx c_1 f(x_1) + c_2 f(x_2)$$

is exact for $f(x) = 1$, x, x^2 and x^3. (the resulting system of four equations has been obtained in class). Then show that the obtained formula has degree of precision 3 (you just need to choose $f(x) = x^4$ and check that the approximation no longer gives the exact integral for this polynomial).

[4] Use the result from [3] and change of variable to derive a quadrature formula for $\int_a^b f(x) dx$ of the same form.

[5] Find constants a, b, c and d that that will produce a quadrature formula

$$\int_{-1}^1 f(x) dx \approx a f(-1) + b f(1) + c f'(-1) + d f'(1)$$

that has the degree of precision three.