Math 151a: HW #7, due on Friday, May 25

[**1**] The forward-difference formula can be expressed as

$$f'(x_0) = \frac{1}{h}[f(x_0+h) - f(x_0)] - \frac{h}{2}f''(x_0) - \frac{h^2}{6}f'''(x_0) + O(h^3).$$

Use extrapolation to derive an $O(h^3)$ formula for $f'(x_0)$.

[**2**] Approximate the integral $\int_0^1 x^2 e^{-x} dx$ using the Trapezoidal, Simpson's and Midpoint Rules. Find a bound for the error using the error formula in each case and compare this to the actual error.

[**3**] The Trapezoidal Rule applied to $\int_0^2 f(x)dx$ gives the value 4, and Simpson's Rule gives the value 2. What is $f(1)$?

[**4**] Find the constants c_0, c_1 and x_1 so that the quadrature formula

$$\int_0^1 f(x)dx \approx c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision.

[**5**] Derive Simpson's Rule with error term by using

$$\int_{x_0}^{x_2} f(x)dx = a_0 f(x_0) + a_1 f(x_1) + a_2 f(x_2) + k f^{(4)}(\xi).$$

Hint: Find first a_0, a_1 and a_2 from the fact that Simpson's Rule is exact for $f(x) = x^n$ with $n = 1$, 2, and 3. Then find k by applying the integration formula with $f(x) = x^4$.