Math 151a: HW #7, due on Friday, May 25

[1] The forward-difference formula can be expressed as

\[ f'(x_0) = \frac{1}{h} [f(x_0 + h) - f(x_0)] - \frac{h}{2} f''(x_0) - \frac{h^2}{6} f'''(x_0) + O(h^3). \]

Use extrapolation to derive an \( O(h^3) \) formula for \( f'(x_0) \).

[2] Approximate the integral \( \int_0^1 x^2 e^{-x} dx \) using the Trapezoidal, Simpson’s and Midpoint Rules. Find a bound for the error using the error formula in each case and compare this to the actual error.

[3] The Trapezoidal Rule applied to \( \int_0^2 f(x) dx \) gives the value 4, and Simpson’s Rule gives the value 2. What is \( f(1) \)?

[4] Find the constants \( c_0, c_1 \) and \( x_1 \) so that the quadrature formula

\[ \int_0^1 f(x) dx \approx c_0 f(0) + c_1 f(x_1) \]

has the highest possible degree of precision.

[5] Derive Simpson’s Rule with error term by using

\[ \int_{x_0}^{x_2} f(x) dx = a_0 f(x_0) + a_1 f(x_1) + a_2 f(x_2) + k f^{(4)}(\xi). \]

Hint: Find first \( a_0, a_1 \) and \( a_2 \) from the fact that Simpson’s Rule is exact for \( f(x) = x^n \) with \( n = 1, 2, \) and \( 3 \). Then find \( k \) by applying the integration formula with \( f(x) = x^4 \).