Math 151a: HW #6, due on Friday, May 18 or Monday, May 21

[1] Let \( x_0, x_1, \ldots, x_n \) be \( n+1 \) distinct points with given values \( f(x_0), f(x_1), \ldots, f(x_n) \). Let \( P_n \) be the Lagrange interpolating polynomial defined using all these points.

(a) Recall the formulas for the divided differences \( f[x_0] \), \( f[x_0, x_1] \), and \( f[x_0, x_1, x_2] \).

(b) Given \( P_n(x) = f[x_0] + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \ldots + a_n(x-x_0)(x-x_1)(x-x_n)(x-x_{n-1}) \),

use \( P_n(x_1) \) to show that \( a_1 = f[x_0, x_1] \).

(c) Given \( P_n(x) = f[x_0] + f[x_0, x_1](x-x_0) + a_2(x-x_0)(x-x_1) + \ldots + a_n(x-x_0)(x-x_1)(x-x_n)(x-x_{n-1}) \),

use \( P_n(x_2) \) to show that \( a_2 = f[x_0, x_1, x_2] \).

[2] Use Newton’s divided difference formula to construct interpolating polynomials of degree one, two and three for the following data. Approximate the specified value \( f(8.4) \) using each of the polynomials if

\( f(8.1) = 16.94410, \ f(8.3) = 17.56492, \ f(8.6) = 18.50515, \ f(8.7) = 18.82091 \).

[3] For a function \( f \), the forward-divided differences are given by:

\[
\begin{align*}
 x_0 &= 0.0 & f[x_0] = & \hfill \phi \\
 x_1 &= 0.4 & f[x_0, x_1] = & \hfill \phi \\
 x_2 &= 0.7 & f[x_1, x_2] = & \hfill \phi
\end{align*}
\]

\( f[x_0, x_1, x_2] = \frac{50}{7} \)

Determine the missing entries in the table and the corresponding Lagrange polynomial of degree 2.

[4] (a) Use the most accurate three-point formula to determine each missing entry in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.3</td>
<td>-0.27652</td>
<td></td>
</tr>
<tr>
<td>-0.2</td>
<td>-0.25074</td>
<td></td>
</tr>
<tr>
<td>-0.1</td>
<td>-0.16134</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
(b) The data in the table was taken from the function $f(x) = e^{2x} - \cos 2x$. Compute the actual errors, and find error bounds using the error formulas.

[5] Let $f(x) = 3xe^x - \cos x$. Use the following data and the Second Derivative Midpoint Formula to approximate $f''(1.3)$ with $h = 0.1$ and with $h = 0.01$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1.20</th>
<th>1.29</th>
<th>1.30</th>
<th>1.31</th>
<th>1.40</th>
</tr>
</thead>
</table>

Compare your results to $f''(1.3)$. 
