Math 151a: HW #6, due on Friday, May 18 or Monday, May 21

[1] Let $x_0, x_1, ..., x_n$ be $n+1$ distinct points with given values $f(x_0)$, $f(x_1)$,..., $f(x_n)$. Let P_n be the Lagrange interpolating polynomial defined using all these points.

(a) Recall the formulas for the divided differences $f[x_0]$, $f[x_0, x_1]$, and $f[x_0, x_1, x_2]$.

(b) Given

$$\begin{aligned} P_n(x) &= f[x_0] + a_1(x-x_0) + a_2(x-x_0)(x-x_1) \\ &+ a_3(x-x_0)(x-x_1)(x-x_2) + ... + a_n(x-x_0)(x-x_1)...(x-x_{n-1}), \end{aligned}$$

use $P_n(x_1)$ to show that $a_1 = f[x_0, x_1]$.

(c) Given

$$\begin{aligned} P_n(x) &= f[x_0] + f[x_0, x_1](x-x_0) + a_2(x-x_0)(x-x_1) \\ &+ a_3(x-x_0)(x-x_1)(x-x_2) + ... + a_n(x-x_0)(x-x_1)...(x-x_{n-1}), \end{aligned}$$

use $P_n(x_2)$ to show that $a_2 = f[x_0, x_1, x_2]$.

[2] Use Newton's divided difference formula to construct interpolating polynomials of degree one, two and three for the following data. Approximate the specified value $f(8.4)$ using each of the polynomials if

$f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, $f(8.7) = 18.82091$.

[3] For a function f, the forward-divided differences are given by:

$$\begin{array}{llll} x_0 = 0.0 & f[x_0] & & \\ & & f[x_0, x_1] & \\ x_1 = 0.4 & f[x_1] & & f[x_0, x_1, x_2] = \frac{50}{7} \\ & & f[x_1, x_2] = 10 & \\ x_2 = 0.7 & f[x_2] = 6. & & \end{array}$$

Determine the missing entries in the table and the corresponding Lagrange polynomial of degree 2.

[4] (a) Use the most accurate three-point formula to determine each missing entry in the following table:

x	$f(x)$	$f'(x)$
-0.3	-0.27652	
-0.2	-0.25074	
-0.1	-0.16134	
0	0	

(b) The data in the table was taken from the function $f(x) = e^{2x} - \cos 2x$. Compute the actual errors, and find error bounds using the error formulas.

[5] Let $f(x) = 3xe^x - \cos x$. Use the following data and the Second Derivative Midpoint Formula to approximate $f''(1.3)$ with $h = 0.1$ and with $h = 0.01$.

x	1.20	1.29	1.30	1.31	1.40
$f(x)$	11.59006	13.78176	14.04276	14.30741	16.86187

Compare your results to $f''(1.3)$.