Math 151a Lecture 4

Homework #4. Due on Friday, October 30

[**1**] Let $f(x) = \sin(\pi x)$ and $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.6$.

(a) Construct interpolation polynomials of degree at most one and at most two to approximate $f(1.4)$ and find the absolute error.

(b) Use the theorem expressing the error in Lagrange interpolation to find an error bound for the approximations.

[**2**] Show that $\max_{x_j \le x \le x_{j+1}} |g(x)| = \frac{h^2}{4}$, where $g(x) = (x - jh)(x - (j+1)h)$.

[**3**] Use Neville's method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two and three to approximate the following:

$f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$.

[**4**] Let $x_0, x_1, ..., x_n$ be $n+1$ distinct points with given values $f(x_0)$, $f(x_1)$,..., $f(x_n)$. Let P_n be the Lagrange interpolating polynomial defined using all these points.

(a) Recall the formulas for the divided differences $f[x_0]$, $f[x_0, x_1]$, and $f[x_0, x_1, x_2]$.

(b) Given

$$\begin{aligned} P_n(x) &= f[x_0] + a_1(x - x_0) + a_2(x - x_0)(x - x_1) \\ &+ a_3(x - x_0)(x - x_1)(x - x_2) + ... + a_n(x - x_0)(x - x_1)...(x - x_{n-1}), \end{aligned}$$

use $P_n(x_1)$ to show that $a_1 = f[x_0, x_1]$.

(c) Given

$$\begin{aligned} P_n(x) &= f[x_0] + f[x_0, x_1](x - x_0) + a_2(x - x_0)(x - x_1) \\ &+ a_3(x - x_0)(x - x_1)(x - x_2) + ... + a_n(x - x_0)(x - x_1)...(x - x_{n-1}), \end{aligned}$$

use $P_n(x_2)$ to show that $a_2 = f[x_0, x_1, x_2]$.

[**5**] Use Newton's divided difference formula to construct interpolating polynomials of degree one, two and three for the following data. Approximate the specified value $f(8.4)$ using each of the polynomials if

$f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, $f(8.7) = 18.82091$.

[**6**] For a function f, the forward-divided differences are given by:

$$\begin{array}{llll}
x_0 = 0.0 & f[x_0] & & \\
 & & f[x_0, x_1] & \\
x_1 = 0.4 & f[x_1] & & f[x_0, x_1, x_2] = \frac{50}{7} \\
 & & f[x_1, x_2] = 10 & \\
x_2 = 0.7 & f[x_2] = 6. & &
\end{array}$$

Determine the missing entries in the table and the corresponding Lagrange polynomial of degree 2.