Math 151A
HW #1, due on Wednesday, July 1st

[1] Using four-digit rounding arithmetic and rationalizing the numerator, find the most accurate approximations to the roots of the following quadratic equation. Compute the absolute errors and the relative errors.

$$\frac{1}{3}x^2 - \frac{123}{4}x + \frac{1}{6} = 0.$$

[2]

(a) Show that the plynomial nesting technique described in Section 1.2 can also be applied to the evaluation of

$$f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^{x} - 1.99.$$

(b) Use three-digit rounding arithmetic, the assumption that $e^{1.53} = 4.62$, and the fact that $e^{nx} = (e^x)^n$ to evaluate $f(1.53)$ as given in part (a).

(c) Redo the calculation in part (b) by first nesting the calculations.

(d) Compare the approximations in parts (b) and (c) to the true three-digit result $f(1.53) = -7.61$.

[3]

(a) Show that the sequence $p_n = (\frac{1}{10})^n$ converges linearly to $p = 0$.

(b) Show that the sequence $p_n = 10^{-2^n}$ converges quadratically to $p = 0$.

You can use a hand calculator or the Bisection Algorithm posted on the class webpage for the following problems.

[4] Use the Bisection method to find p_3 for $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$.

[5] Find an approximation to $\sqrt{3}$ correct to within 10^{-4} using the Bisection Algorithm (hint: consider $f(x) = x^2 - 3$).

[6] Find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-4} to the solution of $x^3 - x - 1 = 0$ lying in the interval $[1, 2]$. Find an approximation to the root with this degree of accuracy.