

**Math 151a****Homework #4. Due on Wednesday, April 30.**

Reading: Sections 1.2 and 3.1

[1] Using four-digit rounding arithmetic and rationalizing the numerator, find the most accurate approximations to the roots of the following quadratic equation. Compute the absolute errors and the relative errors.

$$\frac{1}{3}x^2 - \frac{123}{4}x + \frac{1}{6} = 0.$$

[2]

(a) Show that the polynomial nesting technique described in Section 1.2 can also be applied to the evaluation of

$$f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^x - 1.99.$$

(b) Use three-digit rounding arithmetic, the assumption that  $e^{1.53} = 4.62$ , and the fact that  $e^{nx} = (e^x)^n$  to evaluate  $f(1.53)$  as given in part (a).

(c) Redo the calculation in part (b) by first nesting the calculations.

(d) Compare the approximations in parts (b) and (c) to the true three-digit result  $f(1.53) = -7.61$ .

[3] Let  $f(x) = \sin(\pi x)$  and  $x_0 = 1$ ,  $x_1 = 1.25$ , and  $x_2 = 1.6$ . Construct interpolation polynomials of degree at most one and at most two to approximate  $f(1.4)$ , and find the absolute error. Use the theorem expressing the error, to find an error bound for the approximations.

[4] Show that  $\max_{x_j \leq x \leq x_{j+1}} |g(x)| = \frac{h^2}{4}$ , where  $g(x) = (x - jh)(x - (j+1)h)$ .