Math 151a
Homework #4. Due on Wednesday, April 30.

Reading: Sections 1.2 and 3.1

[**1**] Using four-digit rounding arithmetic and rationalizing the numerator, find the most accurate approximations to the roots of the following quadratic equation. Compute the absolute errors and the relative errors.

$$\frac{1}{3}x^2 - \frac{123}{4}x + \frac{1}{6} = 0.$$

[**2**]

(a) Show that the plynomial nesting technique described in Section 1.2 can also be applied to the evaluation of

$$f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^{x} - 1.99.$$

(b) Use three-digit rounding arithmetic, the assumption that $e^{1.53} = 4.62$, and the fact that $e^{nx} = (e^x)^n$ to evaluate $f(1.53)$ as given in part (a).

(c) Redo the calculation in part (b) by first nesting the calculations.

(d) Compare the approximations in parts (b) and (c) to the true three-digit result $f(1.53) = -7.61$.

[**3**] Let $f(x) = \sin(\pi x)$ and $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.6$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(1.4)$, and find the absolute error. Use the theorem expressing the error, to find an error bound for the approximations.

[**4**] Show that $\max_{x_j \le x \le x_{j+1}} |g(x)| = \frac{h^2}{4}$, where $g(x) = (x - jh)(x - (j+1)h)$.