Math 151a
Homework #4. Due on Wednesday, April 30.

Reading: Sections 1.2 and 3.1

[1] Using four-digit rounding arithmetic and rationalizing the numerator, find the most accurate approximations to the roots of the following quadratic equation. Compute the absolute errors and the relative errors.

\[
\frac{1}{3}x^2 - \frac{123}{4}x + \frac{1}{6} = 0.
\]

[2]
(a) Show that the polynomial nesting technique described in Section 1.2 can also be applied to the evaluation of

\[
f(x) = 1.01e^{4x} - 4.62e^{3x} - 3.11e^{2x} + 12.2e^x - 1.99.
\]

(b) Use three-digit rounding arithmetic, the assumption that \(e^{1.53} = 4.62\), and the fact that \(e^{nx} = (e^x)^n\) to evaluate \(f(1.53)\) as given in part (a).

(c) Redo the calculation in part (b) by first nesting the calculations.

(d) Compare the approximations in parts (b) and (c) to the true three-digit result \(f(1.53) = -7.61\).

[3] Let \(f(x) = \sin(\pi x)\) and \(x_0 = 1, x_1 = 1.25, \) and \(x_2 = 1.6\). Construct interpolation polynomials of degree at most one and at most two to approximate \(f(1.4)\), and find the absolute error. Use the theorem expressing the error, to find an error bound for the approximations.

[4] Show that \(\max_{x_j \leq x \leq x_{j+1}} |g(x)| = \frac{k^2}{4}\), where \(g(x) = (x - jh)(x - (j + 1)h)\).