Math 151A

HW #3, due on Wednesday, April 23

- Reading: section 2.4.
- The problems below from Section 2.4.

[**1**] Use Newton's method to find solutions accurate to within 10^{-5} for the problem:

$$1 - 4x\cos x + 2x^2 + \cos 2x = 0 \qquad \text{for } 0 \le x \le 1.$$

Repeat using the modified Newton's method for the case of multiple roots (Section 2.4).

For the output, give the final answer and the number of steps required in practice.

[**2**] Show that the sequence $p_n = \frac{1}{n^2}$ converges linearly to $p = 0$.

[**3**] Show that the sequence $p_n = 10^{-2^n}$ converges quadratically to $p = 0$.

[**4**] Suppose p is a zero of multiplicity m of f, where $f^{(m)}$ is continuous on an open interval containing p. Show that the following fixed-point method has $g'(p) = 0$:

$$g(x) = x - \frac{mf(x)}{f'(x)}.$$

[**5**] Suppose that f has m continuous derivatives. Modify the proof of the first theorem from "Multiple Roots", to show that: f has a zero of multiplicity m at p if and only if

$$0 = f(p) = f'(p) = ... = f^{(m-1)}(p) = 0, \quad \text{but } f^{(m)} \neq 0.$$