MATH 115A/3, Fall 2007, Midterm #2
Instructor: L. Vese
Teaching Assistant: A. Cantarero

NAME ______________________________

STUDENT ID # ______________________________

This is a closed-book and closed-note examination.
Calculators are not allowed.
Please show all your work.
Partial credit will be given to partial answers.

There are 6 questions of total 100 points.

Time: 55 minutes.

QUESTION	SCORE
[1]	
[2]	
[3]	
[4]	
[5]	
[6]	
TOTAL	100

Questions

[**1**] Define $T : \mathcal{M}_{2\times 2}(R) \mapsto P_2(R)$ by $T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = (a+b) + (2d)x + bx^2$.

Let $\beta = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ and $\gamma = \{1, x, x^2\}$.

Compute $[T]_\beta^\gamma$.

[**2**] Let V, W and Z be vector spaces, and let $T : V \mapsto W$ and $U : W \mapsto Z$. Prove that if U and T are both 1-to-1 and onto, then $UT = U \circ T$ is also 1-to-1 and onto.

[**3**] (a) Give the definition of an isomorphism.

(b) Let B be an $n \times n$ invertible matrix. Define $\Phi : \mathcal{M}_{n\times n}(F) \mapsto \mathcal{M}_{n\times n}(F)$ by

$\Phi(A) = B^{-1}AB$. Prove that Φ is an isomorphism.

(c) Find Φ^{-1}.

[**4**] Consider the linear transformation $T : P_2(R) \mapsto P_2(R)$, defined by

$T(f(x)) = f(1) + f'(0)x + \big(f'(0) + f''(0)\big)x^2$.

(a) Find all eigenvalues of T with their multiplicities (if possible).

(b) Test T for diagonalizability.

(c) If T diagonalizable, determine an ordered basis β of V such that $[T]_\beta$ is a diagonal matrix.

[**5**] (a) Prove that a linear operator $T : V \mapsto V$ on a finite dimensional space is invertible if and only if zero is not an eigenvalue of T.

(b) Let $T : V \mapsto V$ be an invertible linear operator. Prove that a scalar λ is an eigenvalue of T if and only if λ^{-1} is an eigenvalue of T^{-1}.

[**6**] Prove that $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is not diagonalizable.