A Brief Summary of My Researches

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My research interest is to solve problems from mathematics and physics with simple and clean ideas, and along the way to learn and to find new mathematical structures and theories. Localization techniques in various settings, combining with various subjects of mathematics, have played central roles in my researches. See my ICM lecture [38] for more detailed discussions. I divide my past researches into the following several parts:

(1). The applications of modular invariance, the infinite dimensional symmetry in Kac-Moody algebras and mathematical physics, to geometry and topology. From modular invariance we derived several quite general rigidity, divisibility and vanishing results about certain topological invariants of a manifold.

(2). The use of the explicit expressions of heat kernels on Lie groups to study the geometry and topology of moduli spaces of flat connections on Riemann surfaces, this proves certain conjectural formulas of Witten and gives several new results including general vanishing formulas of intersection numbers.

(3). The understanding of mirror symmetry and counting curves on projective manifolds. My joint work with Lian and Yau solves some very general conjectures about the relationship between the numbers of algebraic curves in projective manifolds and hypergeometric series.

(4). The proof of the Marino-Vafa conjecture on Hodge integrals on moduli spaces of Deligne-Mumford curves, which expresses the generating series of three term Hodge integrals over all genera and any number of marked points in terms of a finite expression of representation of symmetric groups, or Chern-Simons knot invariants. We further set up the mathematical foundation of the theory of topological vertex, which gives the most effective ways to give the closed formulas for the generating series Gromov-Witten invariants of all toric Calabi-Yau manifolds of all genera and all degree in terms of the Chern-Simons invariants. These closed formulas are used to prove the Gopakumar-Vafa conjecture for all toric Calabi-Yau manifolds.

(5). The construction of new complete Kahler metrics on moduli spaces curves with bounded negative holomorphic sectional and Ricci curvature, and bounded geometry. As corollaries we prove the equivalence of all the known classical canonical complete metrics on moduli spaces and Teichmüller spaces, including the proofs of two conjectures of Yau in early 80s about the equivalence of the Teichmuller metric, and the Bergman metric with the Kahler-Einstein metric. We obtained the bounded geometry of these metrics and proved the stability of the logarithmic cotangent bundle of the moduli space of Riemann surfaces. We proved the Weil-Petersson metric and the new metrics are good metrics in the sense of Mumford. More interesting results are being worked out

(6). I have also solved some long-standing open problem about algebraic surfaces by using Weil-Peterson metric and Quillen metric; proved certain localization formulas for non-compact group action; proved a Shafarevich conjecture for odd dimensional Calabi-Yau manifolds; and derived various topological and geometrical results on foliated manifolds; constructed K-theory associated to vertex operator algebras and associative algebras; derived asymptotic expansion of the Bergman kernel; first used the quot-scheme to derive the Hori-Vafa formula; derived the asymptotic expansion of the Bergman kernel from heat kernel method. Some works were done with collaborators. See the following for details. The references are listed in the bibliography of my resume. Most of these papers can be downloaded from my webpage at UCLA.

1 Elliptic Genus

1.1 $\hat{A}$-vanishing theorem for loop spaces

This is a loop space analogue of the Atiyah-Hirzebruch $\hat{A}$-vanishing theorem for group actions and the loop space $\hat{A}$-genus, or the Witten genus. An analogue of the Lawson-Yau's vanishing theorem for non-abelian group action is also derived.The proof involves index theory and certain subtle properties of the Jacobi theta-functions. See [3]

1.2 General vanishing theorems associated to loop groups

These theorems give us new obstructions for group actions on manifolds. See [3]. Their proofs uses the modular invariance of the characters of Kac-Moody algebras in a substantial way.

1.3 General rigidity theorem associated to loop groups

This theorem generalizes the famous Witten rigidity conjectures as proved by Taubes, Bott-Taubes, Hirzebruch, Krichever, Landweber-Stong and Ochanine. Here is the first time that Kac-Weyl character formula came into geometry and topology. See [3].

1.4 General miraculous cancellation formula

By using modular forms I generalized a 12-dimensional formula of Alveraz-Gaume and Witten, which they called the miraculous cancellation formula, to arbitrary dimensions and general vector bundles. Joint with Zhang, I found relations between elliptic genus and other geometric invariants, such as holonomy, the APS eta-invariants and the Rochlin invariants. See [4].

1.5 Mod 2 elliptic genus

I have studied mod 2 elliptic genera and answered a question raised by Witten about mod 2 partition functions as mod 2 modular forms in quantum field theory. See [1].

1.6 Elliptic cohomology

In [7] I described an approach to the geometric construction of elliptic cohomology by using the K-group of infinite dimensional vector bundles. A Riemann-Roch type theorem for such K-group was proved.

1.7 Family rigidity and vanishing theorems

Recently in joint works with Xiaonan Ma and Weiping Zhang, we proved several rigidity and vanishing theorems for the family indices of elliptic operators. Applications of these results to fundamental groups are expected. See papers [26]-[30]

1.8 Elliptic genus and foliation

Joint with Ma and Zhang, we proved certain general rigidity and vanishing theorems of elliptic genus for foliated manifolds. To proceed, we constructed new elliptic operators associated to the foliation. See [33].

The proofs of the above results can be considered as combinations of index theory with modular invariance.

1.9 Elliptic genus and vertex operator algebras

Recently with Dong and Ma we are able to prove a rigidity theorem for vertex operator algebra bundles [37]. This strongly indicates that the geometric construction of elliptic cohomlogy is related vertex operator algebras.

2 Heat Kernel and Moduli Spaces

2.1 Intersection numbers on moduli spaces

By using explicit formulas of heat kernels on Lie groups, the Reidemeister torsion and symplectic geometry, I derived very general formulas for the intersection numbers on the moduli spaces of flat connections over a Riemann surface. As consequences several conjectural formulas derived by Witten by using path-integral method are proved. New vanishing formulas for the intersection numbers are derived by using the new method. In principle these formulas contain all the information needed for the Verlinde formula. Such moduli spaces have been studied from many different point of views in the past 50 years. People have used geometric invariant theory, gauge theory and loop groups. My results are for general compact semi-simple Lie groups. Our method also generalizes to the cases when the moduli spaces are singular, as well as when the Riemann surface has several boundary components. Bismut and Labareu were able to prove the general Verlinde formula by following this line of approach. See [14] and [15].

2.2 Vanishing theorems on moduli spaces

As another consequence our method gives several very general new vanishing theorems about the characteristic numbers of the moduli spaces, which actually follows from the delta function property of heat kernels. Some partial results in this direction for $G = SU(n)$ were previously obtained by Atiyah-Bott and Witten in their well-known papers. See [14], [15].

2.3 Compact Lie groups and finite groups

Our heat kernel method gives a systemic and very simple way to find explicit formulas for the numbers of solutions of equations in finite groups. The same technique applies to the derivation of the push-forward measures for commutator maps between compact Lie groups. See [25].

This heat kernel method can be viewed as a natural extension of the heat kernel proof of the Atiyah-Singer index formula, as proposed by Mckean-Singer. It turns out that Witten's nonabelian localization formula can also be seen naturally from the heat kernel point view.

3 Mirror Principle

3.1 Euler data and the Candelas formula

In my joint works with Lian and Yau, we introduced the general notion of Euler data. These are sequences of equivariant cohomology classes in the linear sigma models, the simplest compactifications of the moduli spaces of holomorphic maps from curves into certain projective manifolds with symmetry. We study in detail the algebraic and geometric properties of the Euler data and their properties under mirror transformations. A key technique involved is the Atiyah-Bott equivariant localization formula. Many examples of Euler data arise from the stable compactifications of the above mentioned moduli spaces. We have a conceptual understanding of the mirror symmetry to compute the characteristic classes on the stable map moduli in terms of hypergeometric series. As one of the corollaries we gave a complete proof of the mirror conjecture which relates the counting series of rational curves in a Calabi-Yau quintic manifold to the hypergeometric series of its mirror, as proposed by Candelas and his collaborators.

3.2 General mirror principle

Our method works for computing general characteristic classes on the moduli spaces of stable maps into projective balloon manifolds, which include toric manifolds and homogeneous manifolds, and many of their submanifolds. So far almost all of the formulas as conjectured by string theorists for counting rational curves can be derived from mirror principle.

3.3 Local mirror symmetry

Particularly interesting is our proof of the local mirror symmetry, which was called geometric engineering by physicists. Seiberg-Witten curves always show up in our computations, as predicted by string theory.

3.4 Counting higher genus curves

Now we are in the process of generalizing our methods to count higher genus curves. Most part of our theory works well for higher genus curves. Some new ideas are needed to prove certain general conjectures from string theory.

See [17], [18], [19], [20], [23] and [31] for the details of the above works. Note that we solved these problems with the classical local to global principles in geometry, as developed in the sixties by Atiyah, Bott and Singer.

4 Mariño-Vafa Conjecture and String Duality

The duality among various physical theories has created many beautiful mathematics including the mirror formula, the Mariño-Vafa formula and the topological vertex theory. We first proved the Mariño-Vafa conjecture, then set up the mathematical foundation of the topological vertex theory which has many interesting corollaries in geometry.

4.1 Mariño and Vafa conjecture on triple Hodge integrals

Mariño and Vafa made a remarkable conjecture about the generating series of certain triple Hodge integrals for all genera and all possible marked points. This infinite generating series can be expressed as a finite summation in terms of Chern-Simons knot invariant. They made the conjecture based on the large N duality between Chern-Simons theory and string theory. In joint works with C.-C. Liu and J. Zhou we proved this conjecture by proving a cut-and-join equation for both the combinatorial expressions and by localization on moduli space of relative stable maps. It is interesting to see the same differential equation came out from combinatorics and from geometry. We used to functorial localization formula to push the computations to projective spaces. See [45] and [46].

As easy consequences we derived several well-known Hodge integral identities, including the λ_g conjecture and some other identity conjectured from string theory. See [55].

4.2 Two partition analogue of the Mariño and Vafa formula

We go further to prove a two partition analogue of the Mariño and Vafa formula. The combinatorial expression is given by the Chern-Simons invariant of Hopf link. The convolution formula derived in the localization is quite interesting and directly gave the solution of the cut-and-join equation. See [54].

By using this formula Zhou proved some conjectural formulas for toric Calabi-Yau manifolds from string theorists.

4.3 A Mathematical theory of topological vertex

Topological vertex theory in string theory was developed by Vafa and his collaborators over the past several years since 1998. It gives the most effective way to compute the generating series of all genera and all degrees of both open and closed Gromov-Witten invariants on toric Calabi-Yau manifolds, through a gluing rule of the vertex data. In a joint work with J. Li, C.-C. Liu and J. Zhou we developed a mathematical theory for the topological vertex.

As corollary Pan proved the Gopakumar-Vafa conjecture for all open toric Calabi-Yau manifolds. See [52].

4.4 Equivariant index and topological string partition functions

By comparing localization computations on both Hilbert schemes or the ADHM moduli of Yang-Mills connections on $\mathbb{C}^2$, and the combinatorial expressions of Gromov-Witten invariants on certain toric Calabi-Yau manifolds from topological vertex, we proved that the generating series of Gromov-Witten invariants of all genera and all degrees can be identified to the generating series of the equivariant indices of elliptic operators on the ADHM moduli. This, not only proves some cases of the Gopakumar-Vafa conjecture, but also proves the duality of gauge theory and string theory. See [49].

5 Canonical Metrics on the Moduli Spaces of Riemann Surfaces

In a series of papers joint with X. Sun and S.-T. Yau we introduced new complete Kahler metrics on the moduli and Teichmuller spaces of Riemann surfaces. We studied in details of the two new complete Kähler metrics, the Ricci metric and the perturbed Ricci metric. See [50], [51] and [53] for details.

5.1 Bounded geometry

We proved that the new metrics we introduced have bounded geometry, and gave the precise asymptotic behaviors of the metric and its curvatures. We proved that the perturbed Ricci metric has bounded negative Ricci and holomorphic sectional curvature. This is the first known such metric on the moduli spaces. We proved that the Kahler-Einstein metric has strongly bounded geometry. As consequence we proved that the log cotangent bundle of the moduli spaces of Riemann surfaces of genus larger than 1 are Mumford stable.

Another easy corollary is that all of the classical complete metrics on the moduli and the Teichmuller spaces are equivalent to the two new metrics we introduced.

5.2 Good metrics on moduli spaces

We have proved that the Weil-Petersson metric is a good metric in the sense of Mumford, so are the new metrics we introduced. From these we can derive geometric interesting geometric corollaries by applying L^2-index theory on the Teichmuller and moduli spaces.

6 Other Works

6.1 Equivariant cohomology

In [13] I derived a localization formula which relates different fixed point components and used it to study the moduli spaces of flat connections on a Riemann surface, the Verlinde formula and non-abelian localizations in symplectic geometry.

In [9] I studied equivariant cohomology for non-compact holomorphic or meromorphic group actions on complex manifolds. Several localization and the Duistermaat-Heckman type formulas were obtained.

In [32], we use the Atiyah-Bott-Singer fixed point formula to derive the generating series of the Hirzebruch χ_y-genus for the Hilbert schemes of any complex surface. We are now trying to compute the elliptic genera of Hilbert schemes with this method which will prove a conjecture of string theorists.

6.2 Geometric height inequalities

By using Yau's Schwarz lemma and the Weil-Peterson geometry of the moduli spaces of curves, I obtained quite general geometric height inequalities, the simplest of which implies the functional field analogue of the Mordell conjecture. One of the results was used to solve a conjecture of Beauville about the minimal number of singular fibers for stable fibration over CP^1, and another long-standing conjecture about the strict Chern number inequality of general Kodaira surfaces, which, combining with a result of Yau, implies that the Kodaira surfaces can not be covered by a ball. See [8], [16].

6.3 Foliation

This is joint work with Zhang. By using adiabatic limit method and index theory, we studied foliations and obtained some vanishing theorems for the characteristic numbers of foliations. Particularly interesting is that we derived the Bott connection as the adiabatic limit of a Riemannian connection. See [12].

6.4 Shafarevich conjecture on Calabi-Yau

Joint with Todorov, Yau and Zuo we prove the Shafarevich conjecture about the finiteness of certain families of odd dimensional Calabi-Yau manifolds. See [21].

6.5 On the asymptotic expansions of the Bergman kernel

Joint with X. Dai and X. Ma we give an asymptotic expansion of the Bergman kernel by using heat kernel techniques. The motivation is to extend the result of Donaldson on stability and constant scalar curvature to orbifolds, and to prove a conjecture to relate stability to the identity of the Bergman kernel. See [48] and [56].

6.6 On K-theory associated to vertex operator algebras

Joint with C. Dong, X. Ma and J. Zhou we construct a K-theory associated to the modules of a vertex operator algebra, as well as a K-theory associated to any associative algebras. The motivation is to understand elliptic cohomology. See [44].

6.7 Hori-Vafa formula

we first found the right approach to the Hori-Vafa conjectural formulas for expressions of the basic hypergeometric series by using the quot-scheme compactification and the functorial localization formula. See [36].

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- **SERVICE**

  Communications in Analysis and Geometry, Editor-in-Chief.

  Pacific Journal of Mathematics, Editor.

  Quarterly Journal of Pure and Applied Mathematics, Editor-in-Chief.

- **HONORS**

  * The top 10 Advancements in Science and Technology, 2004, The Ministry of Education of China.

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  * Jian-Qing Zhong Award for the Best Young Mathematicians in China, 1988.

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- **PERSONAL**

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- **TALKS**

  * Invited Speaker at the 2005 Journal of Differential Geometry Conference in Memory of Chern, Boston; at 2004, Yamabe Memorial Conference; at the 2004 International Conference in Several Complex Variables, Beijing; at ICM 2002; at the Shanghai International Algebraic Geometry Conference 2002; at ICCM 2001; at the Wisconsin Orbifold String Theory Conference 2001; at the Southwest Topology Conference 2001.

  * Invited Speaker at the Texas Topology Conference 2000; at the Southwest Algebraic Geometry Conference 2000; at the UCSB Mathematical Physics Conference 1999; at the first ICCM 1998; at the Current Developement in Math. organized by Harvard and MIT 1998; at

the Conference in honor of S.-S. Chern at MSRI 1998; in the Workshop on Elliptic Cohomology in MPI, Bonn 1995; in the Montreal workshop in Mirror Symmetry and Complex Geometry, 1995; at the Geometry and Analysis conferences in Taiwan, 1996, in Hong Kong.

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- **BIBLIOGRAPHY**

1. On Mod 2 and Higher Elliptic Genera. In *Comm. in Math. Physics* Vol.149, No. 1 1992 71-97.
2. On Elliptic Genera and Theta-Functions. In *Topology* Vol. 35, No. 3, 617-640.
3. On Modular Invariance and Rigidity Theorems. In *J. of Diff. Geom.* Vol. 41 1995 343-396.
4. On Modular Invariance and Characteristic Numbers. In *Comm. in Math. Physics,* 174, 29-42 (1995)
5. On $SL_2(Z)$ and Topology. In *Math. Research Letters,* Vol. 1, No. 1, 53-64, 1994.
6. Elliptic Genus and η-invariant (with Zhang). In *Int. Math. Research Notices,* No. 8, 319-328, (1994).
7. Modular Forms and Topology. In *Proc. of the AMS Conference on the Monster and Related Topics,* Contemporary Math., Vol. 193, 237-262 (1996).
8. Remarks on the Geometry of Moduli Spaces. In *Proc. of AMS.* Vol. 124, No. 3 (1996).
9. Holomorphic Equivariant Cohomology. In *Math. Ann.* 303, 125-148 (1995).
10. On the Rigidity of Higher Elliptic Genera (with Gong). In *Ann. Global Analysis.*
11. Remarks on Non-Abelian Localization. In *Int. Math. Research Notices* 1995, No. 13, 683-491.
12. Adiabatic Limit and Vanishing Theorems for Foliations (with Zhang). To appear in Milgram Symposium.
13. Relations among Fixed Points. In *Proc. of Montreal Workshop on Mirror Symmetry*, 1996.
14. Heat Kernel and Moduli Spaces. In *Math. Research Letter* 3, 743-762 (1996).
15. Heat Kernel and Moduli Spaces II. *Math. Research Letter* 4, 569-588 (1997).

16. Geometric Height Inequalities. In *Math. Research Letter* 3, 693-702 (1996).
17. The Candelas-de la Ossa-Green-Parkes Formula. Joint with B. Lian and S.-T. Yau. In String theory, gauge theory and quantum gravity, (Trieste, 1997), *Nuclear Phys. B Proc. Suppl.* 67 (1998), 106-114.
18. Mirror Principle I. Joint with B. Lian and S.-T. Yau. In *Asian J. Math.* 1 (1997), no. 4, 729-763.
19. Mirror Principle II. Joint with B. Lian and S.-T. Yau. In *Asian J. Math.* 3 (1999), no. 1, 109-146..
20. Mirror Principle, A Survey. *Proc. of Current Developement in Math.* 1998.
21. On Shafarevich Conjecture for Odd Dimensional CY Manifolds. Joint with A. Todorov, S.-T. Yau, K. Zuo, *Quarterly Journal of Pure and Applied Math,* 1, 2005.
22. On a Problem of Kuiper. *Science in China*, 2001.
23. Mirror Principle III. Joint with B. Lian and S.-T. Yau. *Asian J. Math.* (1999).
24. On Orbifold Elliptic Genus. Joint with C. Dong and X. Ma. Preprint, 2000.
25. Heat Kernel, Symplectic Geometry, Moduli Space and Finite Groups. *Survey in Differential Geometry,* 1999.
26. Family Rigidity Theorems I. Joint with X. Ma. *Duke Math. J.*, 1999.
27. Family Rigidity Theorems II. Joint with X. Ma. *Montreal Workshop in Mirror Symmetry.*
28. Towards a Mirror Principle for Higher Genus. Joint with B. Lian and S.-T. Yau. *Proceedings of the International Conference in Math., ,Hangzhou 2001.*
29. K-Theory and Family Rigidity Theorems I. Joint with X. Ma, W. Zhang. *Communications in Math. and Physics,* 2000.
30. K-Theory and Family Rigidity Theorems II. Joint with X. Ma, W. Zhang. *Asian Journal of Math.*, 2000.

31. Mirror Principle IV. Joint with Lian and Yau. *Survey in Differential Geometry,* 1999

32. Hirzebruch χ_y genus of Hilbert schemes from fixed point formula. Joint with C. Yan and J. Zhou. *Science in China*, 2001.

33. Elliptic Genus and Foliation. Joint with X. Ma and W. Zhang. *Math. Research Letters.* 2001.

34. A Survey of Mirror Principle. Joint with B. Lian and S.-T.Yau. *Montreal Workshop on Mirror Symmetry and Related Topics*, 2000.

35. Rigidity and Vanishing Theorems in K-theory. *C. R. Acad. France.* 1999.

36. The S^1 fixed points in Quot-schemes and mirror principle computations. Joint with B. Lian, C.-.H Liu and S.-.T Yau, *Comtemp. Math.*, 322, 2003, 165-194.

37. Elliptic genus and vertex operator algebras. Joint with Dong C., X. Ma, math.DG/0201135.

38. Mathematical Results Inspired by Physics. *Proceedings of ICM 2002.*

39. Some applications of mirror principle. Joint with B. Lian and S.-T. Yau, *Contemp. Math.*, Vol 314, 2002.

40. On family rigidity theorem for spin-c manifolds. Joint with X. Ma, *Mirror Symmetry IV*, AMS/IP Stud. Adv. Math., 33, 2002, 343-360.

41. On orbifold elliptic genus. Joint with C. Dong and X. Ma, *Contemp. Math.*, 310, 2002, 87-105.

42. Towards a mirror principle for higher genus. Joint with B. Lian and S.-T. Yau, *Geometry and PDE.* AMS/IP Stud. Adv. Math., 2002, 17-26.

43. Rigidity and vanishing theorems in K-theory I. Joint with X. Ma and W. Zhang, *Commun. Analysis and Geometry*, 11, 2002, 121-180.

44. K-theory associated to vertex operator algebras. Joint with C. Dong, X. Ma, J. Zhou, *Math. Res. Letter*, 11, 629-647(2004).

45. A proof of the Marino-Vafa conjecture on Hodge integrals. Joint with C.-C. Liu and J. Zhou, *J. Diff. Geom.*, 65, 2003, 289-340.

46. On a proof of a conjecture of Marino-Vafa on Hodge integrals. Joint with C.-C. Liu and J. Zhou, *Math. Res. Letter*, 11, 2004, 259-272.

47. On A-twisted moduli stack for curves from Witten's gauged linear sigma models. Joint with C.-H. Liu and S.-T. Yau, *Commun. Analysis and Geometry*, Vol 12, No. 1, 233-280, 2004.

48. On the asymptotic expansion of Bergman kernel. Joint with X. Dai and X. Ma, *C. R. Acad. Sci. Paris, Ser. I* 339(2004), 193-198.

49. Topological String Partition Functions as Equivariant Indices. Joint with J. Li and J. Zhou, math.AG/0412089.

50. Geometric Aspects of the Moduli Space of Riemann Surfaces. Joint with X. Sun, S.-T. Yau, math.DG/0411247, *Proceedings of the International Conference of Complex Geometry,* 2004, Beijing.

51. Canonical Metrics on the Moduli Space of Riemann Surfaces II. Joint with X. Sun, S.-T. Yau, math.DG/0409220. To appear in JDG.

52. A Mathematical Theory of the Topological Vertex. Joint with J.Li, C.-C.Liu and J. Zhou, math.AG/0408426.

53. Canonical Metrics on the Moduli Space of Riemann Surfaces I. Joint with X. Sun, S.-T. Yau, math.DG/0403068. To appear in JDG.

54. A Formula of Two-Partition Hodge Integrals. Joint with C.-C. Liu, J. Zhou, math.AG/0310272.

55. Mariño-Vafa Formula and Hodge Integral Identities. Joint with C.-C. Liu, J. Zhou, math.AG/0308015.

56. On the asymptotic expansion of Bergman kernel. Joint with X. Dai, X. Ma, math.DG/0404494.

57. S^1-fixed-points in hyper-Quot-schemes and an exact mirror formula for flag manifolds from the extended mirror principle diagram. Joint with C.-H. Liu and S.-T. Yau, math.AG/0401367.