

First Name: \_\_\_\_\_

Last Name: \_\_\_\_\_

Section: \_\_\_\_\_

There are **THIRTEEN** problems; five points per problem.**Rules.**

- No calculators, computers, notes, books, crib-sheets,...
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,...
- Try to sit still.
- The answers do not involve nasty integrals; if you arrive at one, check your work.

1	2	3	4	5
6	7	8	9	10
11	12	13	$\Sigma$	

(1) Determine the surface area of the paraboloid

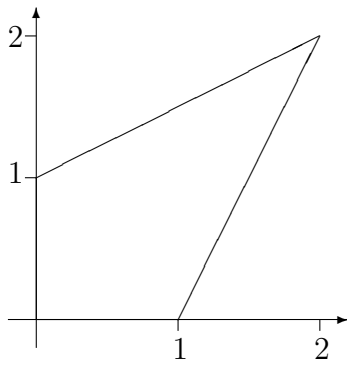
$$x^2 + y^2 = 2z, \quad 0 \leq z \leq 1$$

by whatever means you wish.

(2) Calculate the integral

$$\int_R x^2 dA$$

where  $R$  is the region below:



(3) State the Fundamental Theorem for Line integrals

(4)+(5) Compute both sides of the Divergence Theorem for the cylinder

$$x^2 + y^2 \leq 1 \quad 0 \leq z \leq 1$$

with  $\mathbf{F} = z \sin(x^2 + y^2)\mathbf{k}$ . (Of course, they should turn out to be equal).

[Extra space for (4)+(5)]

(6) Determine

$$\int_R z \, dV$$

where  $R$  is the intersection of the cone  $x^2 + y^2 \leq z^2$  and the unit ball  $x^2 + y^2 + z^2 \leq 1$ .

(7) Justify the following statement: *If  $\nabla \times \mathbf{F} = 0$  then*

$$\oint_{\gamma} \mathbf{F} \cdot d\mathbf{r} = 0$$

*for any closed loop  $\gamma$ .*

(8) Consider the region  $R$  given by

$$0 \leq z \leq (y - x^2)^2, \quad x^2 \leq y \leq x.$$

Use the change of variables

$$x = u, \quad y = v + u^2, \quad z = wv^2,$$

to evaluate

$$\int_R \frac{dV}{y - x^2}$$

(9) Let  $R$  be the region where  $0 \leq y \leq x$  and  $x^2 + y^2 \geq 1$ . Evaluate

$$\int_R \frac{dA}{(x^2 + y^2)^2}$$

by switching to polar coordinates.

(10) Which of the following is conservative:

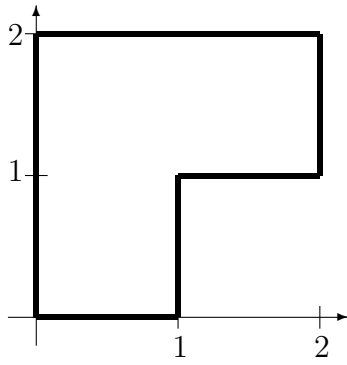
$$\mathbf{F} = x \mathbf{i} + e^y \mathbf{j} + xe^y \mathbf{k} \quad \text{or} \quad \mathbf{F} = ye^x \mathbf{i} + e^x \mathbf{j} + z \mathbf{k}$$

Write it as  $\nabla f$ .

(11) Evaluate

$$\int_C \cos(\pi x) dx + x dy$$

where  $C$  is the curve



(12) Determine

$$\int \frac{dS}{\sqrt{x^2 + z^2}}$$

over the oblique cone parameterized by

$$x = v + v \cos(u), \quad y = v \sin(u), \quad z = v$$

for  $u \in [0, 2\pi]$  and  $v \in [0, 1]$ .

(13) Compute the following integral by reversing the order

$$\int_0^1 \int_{y^2}^y \frac{1}{1 - \sqrt{x}} dx dy$$