32B Killip Practice Final

First Name: ____________________

Last Name: ____________________

Section: ____________________

There are **THIRTEEN** problems; five points per problem.

Rules.

- No calculators, computers, notes, books, crib-sheets,...
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,...
- Try to sit still.
- The answers do not involve nasty integrals; if you arrive at one, check your work.

1	2	3	4	5
6	7	8	9	10
11	12	13	Σ	

(1) Determine the surface area of the paraboloid

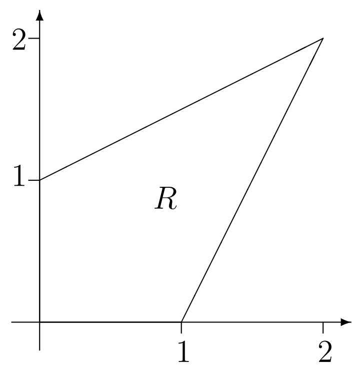
$$x^2 + y^2 = 2z, \qquad 0 \leq z \leq 1$$

by whatever means you wish.

(2) Calculate the integral

$$\int_R x^2 \, dA$$

where R is the region below:



(3) State the Fundamental Theorem for Line integrals

(4)+(5) Compute both sides of the Divergence Theorem for the cylinder

$$x^2 + y^2 \leq 1 \qquad 0 \leq z \leq 1$$

with $\mathbf{F} = z\sin(x^2 + y^2)\mathbf{k}$. (Of course, they should turn out to be equal).

[Extra space for (4)+(5)]

(6) Determine

$$\int_R z\, dV$$

where R is the intersection of the cone $x^2 + y^2 \leq z^2$ and the unit ball $x^2 + y^2 + z^2 \leq 1$.

(7) Justify the following statement: *If* $\nabla \times \mathbf{F} = 0$ *then*

$$\oint_\gamma \mathbf{F} \cdot d\mathbf{r} = 0$$

for any closed loop γ.

(8) Consider the region R given by

$$0 \le z \le (y - x^2)^2, \qquad x^2 \le y \le x.$$

Use the change of variables

$$x = u, \qquad y = v + u^2, \qquad z = wv^2,$$

to evaluate

$$\int_R \frac{dV}{y - x^2}$$

(9) Let R be the region where $0 \leq y \leq x$ and $x^2 + y^2 \geq 1$. Evaluate

$$\int_R \frac{dA}{(x^2 + y^2)^2}$$

by switching to polar coordinates.

(10) Which of the following is conservative:

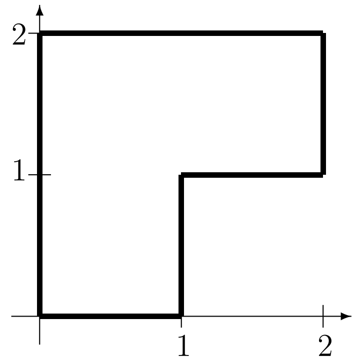
$$\mathbf{F} = x\,\mathbf{i} + e^y\mathbf{j} + xe^y\,\mathbf{k} \qquad \text{or} \qquad \mathbf{F} = ye^x\,\mathbf{i} + e^x\,\mathbf{j} + z\mathbf{k}$$

Write it as ∇f.

(11) Evaluate

$$\int_C \cos(\pi x)\,dx + x\,dy$$

where C is the following loop traversed clockwise



(12) Determine

$$\int \frac{dS}{\sqrt{x^2 + z^2}}$$

over the oblique cone parameterized by

$$x = v + v\cos(u), \quad y = v\sin(u), \quad z = v$$

for $u \in [0, 2\pi]$ and $v \in [0, 1]$.

(13) Compute the following integral by reversing the order

$$\int_0^1 \int_{y^2}^{y} \frac{1}{1-\sqrt{x}}\, dx\, dy$$