32B Lect. 1 Midterm 2 February 28

First Name: ____________________ ID#____________________

Last Name: ____________________

Section: ____________________ $= \begin{cases} 1a & \text{Tuesday with Eric Auld} \\ 1b & \text{Thursday with Eric Auld} \\ 1c & \text{Tuesday with Kyung Ha} \\ 1d & \text{Thursday with Kyung Ha} \\ 1e & \text{Tuesday with Khang Huynh} \\ 1f & \text{Thursday with Khang Huynh} \end{cases}$

Rules.

- There are **FOUR** problems; ten points per problem.
- There are two extra pages at the end. You may also use the backs of pages.
- No calculators, computers, notes, books, crib-sheets,...
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,... Try to sit still.
- Turn off your cell-phone.

1	2	3	4	$\sum$

(1) Let $\mathcal{R}$ be the region in the plane where $x^2 + y^2 \geq 4$. Evaluate

$$\iint_{\mathcal{R}} \frac{1}{(x^2+y^2)\ln^2(x^2+y^2)}\, dA$$

by the change of variables $x = e^u \cos(v)$, $y = e^u \sin(v)$. As usual, $\ln^2(w)$ denotes the square of the natural logarithm of w.

(2) I promise that the vector field

$$\vec{F}(x, y, z) = \begin{bmatrix} 3y + z^2 \\ 3x \\ 2xz \end{bmatrix}$$

is conservative.

(a) Find a potential $V(x, y, z)$ so that $\vec{F} = -\nabla V$.

(b) Compute the line integral $\displaystyle\int_\gamma \vec{F} \cdot d\vec{r}$ where γ is the path parameterized by

$$\vec{r}(t) = \begin{bmatrix} 1 - t \\ t^3 \\ (1 - t)^2 \end{bmatrix} \qquad 0 \leq t \leq 1$$

(3) (a) Let $\mathcal{R}$ be the region in the plane where $x^2 + y^2 > 1$. Is $\mathcal{R}$ simply connected?

(b) Let $\mathcal{Q}$ be the region in the plane where $x^2 + y^2 > 1$ and $x > -\frac{1}{2}$. Is $\mathcal{Q}$ simply connected?

(c) Does the following vector field obey the mixed-partials condition for begin conservative?

$$\vec{F}(x,y) = \frac{1}{1 - x^2 - y^2} \begin{bmatrix} x \\ y \end{bmatrix}$$

(d) Evaluate the line integral $\displaystyle\int_\gamma \|\vec{F}\| \, ds$ where γ is the straight path from $(2,0)$ to $(4,0)$.

(4) Compute the flux of the vector field

$$\vec{F} = \begin{bmatrix} y \\ 0 \\ z \end{bmatrix}$$

upwards through the rectangle with corners $(0, 3, 0)$, $(1, 3, 0)$, $(1, 0, 2)$ and $(0, 0, 2)$. Here 'upwards' means in the direction of the positive z-axis.

$\boxed{\text{extra paper}}$

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