32B Killip Mid-term 1 Oct 17

First Name: _____ ID#_____

Last Name: _____

Section: _____ $= \begin{cases} 1a & \text{Tuesday with Keegan} \\ 1b & \text{Thursday with Keegan} \\ 1c & \text{Tuesday with Tserunyan} \\ 1d & \text{Thursday with Tserunyan} \\ 1e & \text{Tuesday with Kwon} \\ 1f & \text{Thursday with Kwon} \end{cases}$

Midterm rules.

- There are five problems; five points per problem.
- No calculators, computers, notes, books, crib-sheets,...
- This is a **50 minute** exam.
- Out of consideration for your class-mates, no chewing, humming, pen-twirling, snoring,... Try to sit still.
- Turn off your cell-phone, pager,...
- The answers do not involve difficult integrals; if you arrive at one, check your work.

1	2	3	4	5	Σ

(1) Determine the volume of the region given by $0 \leq x \leq 1$, $0 \leq y \leq 2$, and $-x \leq z \leq xy$.

(2) Evaluate

$$\int_0^1 \int_x^1 \sin(y^2)\, dy\, dx$$

by reversing the order of integration.

(3) Re-write the following integral in polar coordinates:

$$\int_0^{\sqrt{3}} \int_{x^2}^{\sqrt{12-x^2}} \frac{x^2}{(x^2+y^2)^{3/2}}\, dy\, dx$$

DO NOT compute the value of this integral.
Hint: $\arccos(\frac{1}{2}) = \arctan(\sqrt{3}) = \arcsin(\frac{1}{2}\sqrt{3}) = \frac{\pi}{3}$.

(4) Consider the solid pyramid given by

$$0 \leq |x| \leq y \leq 1 \quad \text{and} \quad 0 \leq |z| \leq y \leq 1$$

If its mass density is $\rho(x, y, z) = y + z$, determine the total mass by writing it as a triple integral.

(5) Evaluate the following triple integral:

$$\int_0^{2\pi} \int_0^1 \int_0^r \frac{r \; dz \; dr \; d\theta}{r^2 + z^2}$$

by converting from cylindrical to spherical coordinates.

extra paper