(1) We take two cards (without replacement) from a well-shuffled standard deck of 52 cards. Let X denote the number of these two cards that are aces and let Y denote the number that are hearts.
(a) Tabulate the joint PMF for X and Y.
(b) Compute the PMF for Y both directly and as a marginal of the above (this provides a check on your computations).
(c) What is the covariance of X and Y?

(2) Each of n people (whom we label 1, 2, $\ldots$, n) are randomly and independently assigned a number from the set $\{1, 2, 3, \ldots, 365\}$ according to the uniform distribution. We will call this number their birthday.
(a) Describe a sample space Ω for this scenario.
Let j and k be distinct labels (between 1 and n) and let A_{jk} denote the event that the corresponding people share a birthday. Let X_{jk} denote the indicator random variable associated to A_{jk}.
(b) Write A_{12} as a subset of Ω.
(c) Tabulate the joint PMF for X_{12} and X_{13}. Compute the PMF for the product $X_{12}X_{13}$.
(d) Tabulate the joint PMF for X_{12} and X_{34}. Compute the PMF for the product $X_{12}X_{34}$.
(e) Are A_{12} and A_{34} independent? Are they independent conditioned on A_{13}?
(f) Are A_{12} and A_{13} independent? Are they independent conditioned on A_{23}?
(g) Compute the expected number of pairs of people who share a birthday (hint: write this the number as a sum of X_{jk}s).
(h) Compute the second moment and variance of the number of pairs of people who share a birthday.

(3) My dryer contains three pairs of socks of different colors. I blindly draw socks from the dryer one at a time until I have a matching pair; let X denote the number of socks taken from the dryer when this happens. Describe this experiment with a tree. Compute the PMF, mean, and variance of X.

(4) A student answers a True/False quiz with twenty questions by tossing a coin. What is the PMF, mean, and variance of the number of correct answers.