(1) Each of n people are randomly and independently assigned a number from the set $\{1, 2, 3, \dots, 365\}$ according to the uniform distribution. We will call this number their birthday.
(a) What is the probability that no two people share a birthday?
(b) Use a computer or calculator to evaluate your answer as a decimal for $n = 22$ and $n = 23$.

(2) Suppose $X \sim \text{Poisson}(\lambda)$. By repeatedly differentiating the power series for e^λ, compute

$$\mathbb{E}\big\{X(X-1)(X-2)\cdots(X+1-\ell)\big\}$$

for each $\ell = 1, 2, 3, \dots$.

(3) Suppose $X \sim \text{Binomial}(n, p)$. (a) By repeatedly differentiating the binomial identity compute

$$\mathbb{E}\big\{X(X-1)(X-2)\cdots(X+1-\ell)\big\}$$

for each $\ell = 1, 2, 3, \dots$.
(b) If $n \to \infty$ with $p = \lambda/n$ show that your answer converges to the answer to the preceding question.

(4) Suppose $X \sim \text{Poisson}(\lambda)$. Compute the probability that X is even.

(5) We deal seven cards from a well-shuffled standard deck of 52 cards. Compute the PMF, mean, and variance of the number of aces dealt.

(6) We throw a die independently four times and let X denote the minimal value rolled.
(a) What is the probability that $X \geq 4$.
(b) Compute the PMF of X.
(c) Determine the mean and variance of X.

(7) Compute $\mathbb{E}(e^{tX})$ as a function of $t \in \mathbb{R}$ when
(a) $X \sim \text{Poisson}(\lambda)$,
(b) $X \sim \text{Binomial}(n, p)$.

(8) Problem 20 from Chapter 2.