

- (1) I repeatedly attempt the same task. My probability of success, on the k^{th} attempt is $p \in (0, 1)$, irrespective of the outcomes of all previous attempts.
 - (a) Draw the first few levels of a tree description of this experiment.
 - (b) What is the probability that my first success occurs on an odd numbered attempt.
- (2) I repeatedly attempt the same task. My probability of success, on the k^{th} attempt is $k(k+2)/(k+1)^2$, irrespective of the outcomes of all previous attempts. Here $k \in \{1, 2, 3, \dots\}$. What is the probability that I never fail?
- (3) A stack of 26 cards is chosen from a standard deck of 52 playing cards in a completely random manner. Compare (determine $<$, $>$, or $=$) the probabilities of the following: (a) the stack contains all the clubs; and (b) the stack contains no clubs at all.
- (4) A bag contains either a blue or yellow counter with either possibility equally likely. A yellow counter is added and the bag well shaken. A randomly chosen counter is removed from the bag and turns out to be yellow. What is the probability that the remaining counter is yellow?
- (5) (a) You draw two dominoes at random from a standard “double-six” set (and keep both). What is the probability that none of your dominoes has a 6 on it (at either or both ends).
(b) Starting again, you and your opponent alternately draw dominoes randomly from the set (you start) until you both have two. It is intuitive the probability that none of your dominoes has a 6 on it (at either or both ends) is unchanged. Draw a tree and verify this with formulas.
- (6) A magnetic tape storing information in binary form has been corrupted, so it can no longer be read reliably. The probability that you correctly detect a 0 is 0.9, while the probability that you correctly detect a 1 is 0.85. Each digit is a 1 or a 0 with equal probability. Given that you read a 1, what is the probability that this is a correct reading?
- (7) A crime is committed by a single person on an island with population 50 000. The implement used is only owned by 100 people on the island; we know one of these people is the criminal.
 - (a) Given that someone is not the criminal, what is the probability that they own the implement?
 - (b) Given that someone owns the implement, what is the probability that they are the criminal?
 - (c) (For private consideration) Which probability should be presented to a jury?

Continued...

- (8) Consider a university comprised of two schools, say A and E . We wish to model whether Assistant Professors are promoted to tenure (which we label T) or not (written N) and whether this depends on their gender (M or F). We use the letters A, E, T, N, M, F to represent these events. It happens that the probability that an Assistant Professor is female is $1/2$, that is, $\mathbb{P}(F) = 1/2$.
- (a) Across the university as a whole, females are promoted with probability $1/2$ and males with probability $9/16$. Write these as conditional probabilities.
- (b) The probability of promotion for Assistant Professors in A is $2/5$; in E , it is $11/17$. Write these as conditional probabilities.
- (c) Connect the probability that a female is promoted (university wide) to the probabilities that this happens conditioned on the event that she works in A and the event that she works in E .
- (d) It is claimed that within each school, the probability that a female is promoted is not smaller than the probability that a male is promoted. Write this mathematically.
- (e) Show that everything in this problem is consistent by writing down underlying probabilities of the eight events that reproduce all the statements above as well as the following: the ratio of Assistant professors working in A and E is 15:17 and

$$\mathbb{P}(A \cap T \cap M) = \mathbb{P}(E \cap N \cap F) = 1/16.$$

In particular, compute all four probabilities mentioned in part (d).